Let's Be Rational

Investigation 1

Standards:

6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. *Problem 1*

6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *Problems 3 and 4*

Essential for 6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. *Problems 3 and 4*

Essential for 6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *Problems 3 and 4*

Essential for 6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *Problem 3*

Essential for 6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. *Problems 1 and 2*

Essential for 6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. *Problem 4*

Essential for 6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. *Problems 1, 2, and 4*

Objectives:

Investigation 1 focuses on estimating sums of fractions and decimals, and then moves to finding algorithms for precise computations. It builds on *Comparing Bits and Pieces* by extending skills with benchmarks, equivalent fractions, and the relationship between decimals and fractions.

Students play a game in which they estimate the size of sums. Students also explore estimation as a strategy to reason about contextual situations. They identify whether their estimate is an underestimate or an overestimate.

This Investigation also focuses on developing computational understanding and skill in adding and subtracting fractions. Students are not given algorithms for computation. Instead, students figure out how to add and subtract fractions by being flexible in finding equivalent fractions. This Investigation serves as an important formative assessment of students' understanding of fractions.

In the course of solving the problems, students develop strategies for adding and subtracting fractions and mixed numbers. Through class discussion, these strategies are made more explicit and efficient.

Investigation 2

Standards:

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *Problems 2 and 3*

Essential for 6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *Problems 1, 2, and 3*

Objectives:

Investigation 2 focuses on developing computational skill with and understanding of fraction multiplication. Various contexts and models are introduced to help students make sense of when multiplication is appropriate.

In Problem 2.1 students develop an understanding of multiplication with simple fractions. Problems 2.2 and 2.3 focus on multiplication with fraction, mixed number, and whole-number combinations.

Estimation is used across the Problems so that students can determine the reasonableness of their answers. Also, students develop the idea that multiplication does not always lead to a larger product. Within these Problems, students form a general algorithm for fraction multiplication.

Investigation 3

Standards:

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *Problems 1, 2, 3, and 4*

6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *Problem 1*

Objective:

Investigation 3 explores the operation of division. Problem 3.1 emphasizes estimation. The context involves measurements that provide a familiar setting for the first interpretation of division. Students

are asked to consider *How much of this is in that?* using fractional dividends and divisors. Everyday situations are used to help students make sense of when division is an appropriate operation. The first two Problems in the Investigation involve the measuring interpretation of division, and the third involves the sharing interpretation. The last Problem uses presorted division problems to develop a general algorithm for fraction division.

Investigation 4

Standards:

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. *Problems 1, 2, and 3*

6.EE.A.2a Write expressions that record operations with numbers and with letters standing for numbers. *Problems 1, 2, and 3*

6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *Problems 1, 2, and 3*

6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *Problems 1, 2, and 3*

6.EE.B.6Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. *Problems 1, 2, and 3*

6.EE.B.7Solve real-world and mathematical problems by writing and solving equations of the form $x+p=q$ and $px=q$ for cases in which p, q and x are all nonnegative rational numbers. *Problems 1, 2, and 3*

Objective:

Investigation 4 concludes *Let's Be Rational*'s exploration of the four arithmetic operations with fractions, first by considering more abstract relationships among the four operations, and second by considering what kinds of contexts lead to each of the operations in order to highlight their structure. Problems 4.1 and 4.2 have students use fact families to express additive and multiplicative relationships among fractional quantities. In doing so, students also work on the ideas of inverse and decomposition, each of which is an important numerical precursor to algebra work. Problem 4.3 presents students with a variety of contextual problems. Students have to sort out for themselves which operations apply in each situation. Being able to recognize which operation is called for in a problem situation strengthens students' understanding of the meaning of each operation.