6-1: Prime Time
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Factors and Multiples Understand relationships among factors, multiples, divisors, and products

- Classify numbers as prime, composite, even, odd, or square
- Recognize that factors of a number occur in pairs
- Recognize situations that call for common factors and situations that call for common multiples
- Recognize situations that call for the greatest common factor and situations that call for the least common multiple
- Develop strategies for finding factors and multiples
- Develop strategies for finding the least common multiple and the greatest common factor
- Recognize and use the fact that every whole number can be written in exactly one way as a product of prime numbers
- Use exponential notation to write repeated factors
- Relate the prime factorization of two numbers to the least common multiple and greatest common factor of two numbers
- Solve problems involving factors and multiples

Equivalent Expressions Understand why two expressions are equivalent

- Relate the area of a rectangle to the Distributive Property
- Recognize that the Distributive Property relates the multiplicative and additive structures of whole numbers
- Use the properties of operations of numbers, including the Distributive Property, and the Order of Operations convention to write equivalent numerical expressions
- Solve problems involving the Order of Operations and Distributive Property

6-1 Prime Time: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Building on Factors and Multiples	Investigation 2 Common Multiples and Common Factors	Investigation 3 Factorizations: Searching for Factor Strings	Investigation 4 Linking Multiplication and Addition: The Distributive Property
Problem 1.1 Playing the Factor Game: Finding Proper Factors FQ: How can you find all the factors (or divisors) of a number? Problem 1.2 Playing to Win: Prime and Composite Numbers FQ: What information about a number can you find by looking at its factors? Problem 1.3 The Product Game: Finding Multiples FQ: If you know one factor of a number, how can you find another factor of the number? Problem 1.4 Rectangles and Factor Pairs FQ: How do you know when you have found all of the factors of a number?	Problem 2.1 Riding Ferris Wheels: Choosing Common Multiples or Common Factors FQ: How can you decide when finding common multiples is useful in solving a problem? Problem 2.2 Looking at Cicada Cycles: Choosing Common Multiples or Common Factors FQ: How can you find the least common multiple of two or more numbers? Problem 2.3 Bagging Snacks: Choosing Common Multiples or Common Factors FQ: How can you decide when finding common factors is useful in solving a problem? How can you find the greatest common factor of two numbers?	Problem 3.1 The Product Puzzle: Finding Factor Strings FQ: How can you find the prime factorization of a number? Problem 3.2 Finding the Longest Factor String FQ: How many unique prime factorizations of a number are there? Problem 3.3 Using Prime Factorizations FQ: How can the prime factorization of a number be used to find the LCM and GCF of two or more numbers? Problem 3.4 Unraveling the Locker Problem: Putting It All Together FQ: What characteristics of numbers, such as factors and multiples did you use to answer the questions? What special numbers, such as prime numbers, composite numbers, and square numbers, did you use?	Problem 4.1 Reasoning With Even and Odd Numbers FQ: How do you decide whether a number is even or odd? Problem 4.2 Using the Distributive Property FQ: How is the Distributive Property used to create equivalent expressions? How is finding the area of a rectangle related to the Distributive Property? Problem 4.3 Ordering Operations FQ: How do you decide the order when you work on number sentences with more than one operation? Problem 4.4 Choosing and Operation FQ: How do you decide what operations are needed in a given situation?
Mathematical Reflections 1a. Explain how factors and multiples of a number are related. 1b. Describe a situation where it is useful to know about factors and multiples. 1c. Describe strategies for finding factors or multiples of a number. 2. You can describe a number by both the number of its factors and the kind of its factors. Describe several kinds of numbers that you studied in this Investigation. Give examples.	Mathematical Reflections 1. How can you decide if finding common multiples or common factors is helpful in solving a problem? Explain. 2a. Describe how you can find the common factors and the greatest common factor of two numbers. 2b. What information does the greatest common factor of two numbers provide in a problem? 3a. Describe how you can find the common multiples and the least common multiple of two numbers. 3b. What information does the least common multiple of two numbers provide in a problem?	Mathematical Reflections 1a. Why it is helpful to write a number as a product of primes? 1b. Describe how you can find the prime factorization of a number. 2a. When it is useful to find the LCM or GCF of two or more numbers to solve a problem? 2b. Describe a method for finding the LCM of two numbers. Is there another method? Explain. 2c. Describe a method for finding the GCF of two numbers. Is there another method? Explain.	Mathematical Reflections 1a. Explain what the Distributive Property means for multiplication, addition, and subtraction. Use the area of a rectangle to illustrate your answer. 1b. Explain how you can use the Distributive Property to write a number as two equivalent expressions. Give two examples. 2a. What rules for ordering computations with numbers does the Order of Operations convention provide? Why is it important? 2b. How do you decide what operation, addition, subtraction, multiplication, or division, is needed to solve a problem?

6-2: Comparing Bits and Pieces
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Fractions as numbers. Understand fractions and decimals as numbers that can be located on the number line, compared, counted, partitioned and decomposed.

- Expand interpretations of a fraction to include, expressing fractions as a part-whole relationship, as a number, and as an indicated division.
- Reason about the roles of numerator and denominator in each of the interpretations of a fraction.
- Use multiple interpretations of proper fractions, improper fractions and mixed numbers.
- Use decimals to represent fractional quantities, with attention to place value.
- Recognize that the set of positive and negative fractions is called rational numbers and recognize rational numbers as points on the number line.
- Use the number line to reason about rational number relationships.
- Use benchmarks to estimate the size of fractions (and decimals), to compare and order fractions (and decimals).
- Recognize that fractions (both positive and negative) can represent both locations and distances on the number line.
- Recognize that a number and its opposite are equal distances from zero on the number line. The opposite of a is –a and the opposite of –a is a.
- Understand that the *absolute value* of a number is its distance from 0 on the number line and use it to describe real-world quantities.
- Introduce percents as a part-whole relationship where the whole is not necessarily out of 100, but is scaled or partitioned to be "out of 100" or “per 100.”
- Apply a variety of partitioning strategies to solve problems.

Ratios as comparisons. Understand ratios as comparisons of two numbers.

- Use ratios and their associated rates to compare quantities.
- Distinguish between difference (additive comparison) and ratio (multiplicative comparison).
- Distinguish between fractions as numbers and ratios as comparisons.
- Apply a variety of scaling strategies to solve problems involving ratios and unit rates.
- Understand that a unit rate is a ratio in which one of the quantities being compared has a value of 1; use rate language in the context of a ratio relationship.
- Scale percents to predict new outcomes.

Equivalence. Understand equivalence of fractions and of ratios, and use equivalence to solve problems.

- Understand that equivalent fractions represent the same amount, distance or location; develop strategies for finding equivalent fractions.
- Understand that comparing situations with different-sized wholes is difficult unless we use some common basis of comparison.
- Use partitioning and scaling strategies to generate equivalent fractions and ratios, and to solve problems.
- Develop meaningful strategies for representing fraction amounts larger than one or less than zero as both mixed numbers and improper fractions.
- Understand that equivalent ratios represent the same relationship between two quantities; develop strategies for finding and using equivalent ratios.
- Build and use rate tables of equivalent ratios to solve problems.

6-2 Comparing Bits and Pieces: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Making Connections	Investigation 2 Connecting Ratios and Rates	Investigation 3 Extending the Number Line	Investigation 4 Working With Percents
Problem 1.1 Fundraising: Comparing With Fractions and Ratios FQ:: What are two ways to compare a $500 fundraising goal to a $200 fundraising goal? Problem 1.2 Fundraising Thermometers: Introducing Ratios FQ: How does a "for every" statement show a ratio comparison? Problem 1.3 Equivalent Fractions on the Line FQ: When you fold fraction strips, what relationships do you see emerge that show how the numerator and denominator change to make equivalent fractions? Problem 1.4 Measuring Progress: Finding Fractional Parts FQ: How can fraction strips help you find part of a number? Problem 1.5 Comparing Fundraising Goals: Using Fractions and Ratios FQ: What does it mean for two fractions to be equivalent? What does it mean for two ratios to be equivalent?	Problem 2.1 Equal Shares: Introducing Unit Rates FQ: What does a unit rate comparison statement tell us? Problem 2.2 Unequal Shares: Using Ratios and Fractions FQ: How are part-to-part relationships related to part-to-whole fractions? Problem 2.3 Making Comparisons with Rate Tables FQ: How do rate tables help us find equivalent ratios?	Problem 3.1 Extending the Number Line: Integers and Mixed Numbers FQ: How can the number line help you think about fractions greater than 1 and less than 0? Problem 3.2 Estimating and Ordering Rational Numbers: Comparing Fractions to Benchmarks FQ: When comparing two relational numbers, what are some useful strategies for deciding which is greater? Problem 3.3 Sharing 100 Things: Using Tenths and Hundredths FQ: How does what you know about fractions help you understand decimals? Problem 3.4 Decimals on the Number Line FQ: How do we use what we know about fractions to estimate and compare decimals? Problem 3.5 Earthquake Relief: Moving from Fractions to Decimals FQ: Why does it make sense to divide the numerator of a fraction by the denominator to find an equivalent decimal representation?	Problem 4.1 Who is the Best? Making Sense of Percents FQ: How is a percent bar useful in making comparisons with decimals? Problem 4.2 Genetic Traits: Finding Percents FQ: How can partitioning be used to express one number as a percent of another number? Problem 4.3 The Art of Comparison: Using Ratios and Percents FQ: In what way is a percent like a ratio and like a fraction?
Mathematical Reflections 1a. Write three comparison statements about the same situation, one using difference, one using a fraction, and one using a ratio. 1b. Explain what you think a ratio is. 2a. What does it mean for two fractions to be equivalent? For two ratios to be equivalent? 2b. What are some useful ways of finding equivalent fractions and equivalent ratios?	Mathematical Reflections 1a. How can you determine a unit rate for a situation? 1b. Describe some ways that unit rates are useful. 2a. What strategies do you use to make a rate table? 2b. Describe some ways that rate tables are useful. 3. How are your strategies for writing equivalent ratios the same as or different from writing equivalent fractions?	Mathematical Reflections 1a. Not every fraction refers to a quantity between 0 and 1. Give some examples of numbers that are greater than 1 or less than 0. 1b. How is a number and its opposite represented on a number line? 2a. What are some strategies for deciding which of two numbers is greater? Give examples. 2b. When comparing two positive whole numbers with different numbers of digits, such as 115 and 37, the one with more digits is greater. Does this rule work for comparing decimals?	Mathematical Reflections 1. Describe strategies for finding a percent of a known quantity. 2. What strategies can you use to find the percent of one quantity to another quantity? 3. How are percents used to make a comparison? 4. Describe other strategies that you can use to make comparisons.

6-3: Let's Be Rational
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Numeric Estimation Understand that estimation can be used as a tool in a variety of situations including checking answers and making decisions, and develop strategies for estimating results of arithmetic operations

Use benchmarks and other strategies to estimate results of operations with fractions
Use estimates to check the reasonableness of exact computations
Give various reasons to estimate and identify when a situation calls for an overestimate or an underestimate
Use estimates and exact solutions to make decisions

Fraction Operations Revisit and continue to develop meanings for the four arithmetic operations and skill at using algorithms for each

Determine when addition, subtraction, multiplication, or division is the appropriate operation to solve a problem
Develop ways to model sums, differences, products, and quotients with areas, fraction strips, and number lines
Use knowledge of fractions and equivalence of fractions to develop algorithms for adding, subtracting, multiplying, and dividing fractions
Write fact families with fractions to show the inverse relationship between addition and subtraction, and between multiplication and division
Compare and contrast dividing a whole number by a fraction to dividing a fraction by a whole number
Recognize that when you multiply or divide a fraction, your answer might be less than or more than the numbers you started with
Solve real-world problems using arithmetic operations on fractions

Variables and Equations Use variables to represent unknown values and equations to represent relationships

Represent unknown real-world and abstract values with variables
Write equations (or number sentences) to represent relationships among real-world and abstract values
Use fact families to solve for unknown values

6-3 Let’s Be Rational: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Extending Addition and Subtraction of Fractions	Investigation 2 Building Multiplication With Fractions	Investigation 3 Dividing With Fractions	Investigation 4 Wrapping Up the Operations
Problem 1.1 Getting Close: Estimating Sums FQ: What are some strategies for estimating the sums of fractions? Problem 1.2 Estimating Sums and Differences FQ: How do you know if your estimate is an underestimate or overestimate? What information does an underestimate or overestimate tell you? Problem 1.3 Land Sections: Adding and Subtracting Fractions FQ: What are some strategies for adding and subtracting fractions? Problem 1.4 Visiting the Spice Shop: Adding and Subtracting Mixed Numbers FQ: What are some strategies for adding and subtracting mixed numbers?	Problem 2.1 How Much of the Pan Have We Sold? Finding Parts of Parts FQ: How does an area model relate to multiplying fractions? Problem 2.2 Modeling Multiplicative Situations FQ: What strategies can you use to multiply all combinations of factors including whole numbers, fractions, and mixed numbers? Problem 2.3 Changing Forms: Multiplication With Mixed Numbers FQ: How can you use number properties and equivalent fractions to multiply rational numbers?	Problem 3.1 Preparing Food: Dividing a Fraction by a Fraction FQ: What does it mean to divide a fraction by a fraction? What strategies help you divide a fraction by a fraction? Problem 3.2 Into Pieces: Whole Numbers or Mixed Numbers Divided by Fractions FQ: What does it mean to divide a whole number or mixed number by a fraction? What strategies help you divide a whole number or mixed number by a fraction? Problem 3.3 Sharing a Prize: Dividing a Fraction by a Whole Number FQ: What does it mean to divide a fraction by a whole number? What strategies help you divide a fraction by a whole number? Problem 3.4 Examining Algorithms for Dividing Fractions FQ: What is an efficient algorithm for division problems involving fractions and mixed numbers?	Problem 4.1 Just the facts: Fact Families for Addition and Subtraction FQ: How do fact families help you solve equations such as $\frac{4}{5} - N = \frac{3}{8}$? Problem 4.2 Multiplication and Division Fact Families FQ: How do fact families help you solve equations such as $\frac{2}{9} \div N = \frac{2}{3}$? Problem 4.3 Becoming an Operations Sleuth FQ: How do you know when a particular operation is called for to solve a problem? How do you represent the problem with a number sentence?
Mathematical Reflections 1a. What are some situations in which estimating a sum or difference is useful? Why is estimation useful in these situations? 1b. When is it useful to overestimate? When is it useful to underestimate? 2. When should you use addition to solve a problem involving fractions? When should you use subtraction? 3. Suppose you are helping a student who has not studied fractions. Explain to him or her how to add and subtract fractions. Give an example of the type you think is easiest to explain. Give an example of the type you think is hardest to explain.	Mathematical Reflections 1. Explain and illustrate what *of* means when you find a fraction *of* another number. What operation do you use when you find parts of parts? 2a. If you forget the algorithm for multiplying fractions, how might you use rectangular models to help you multiply fractions? 2b. Describe an algorithm for multiplying any two fractions. 2c. Describe when it might be useful to estimate a product. 3. Use examples to explain the following statement: “When you multiply a fraction by another fraction, your answer might be less than both factors, more than one of the factors, or more than both factors.”	Mathematical Reflections 1. When solving a problem, how do you recognize when division is the operation you need to use? 2a. How is dividing a whole number by a fraction similar to or different from dividing a fraction by a whole number? 2b. Explain your strategy for dividing one fraction by another fraction. Does your strategy also work for divisions where the dividend or divisor is a whole number or a mixed number? Explain. 3. When dividing a whole number by a whole number greater than 1, the quotient is always less than the dividend. For example, 15 ÷ 3 = 5, and 5 is less than 15 (the dividend). Use examples to explain the following statement: “When you divide a fraction by another fraction, your answer might be greater than the dividend or less than the dividend.”	Mathematical Reflections 1. How do you decide which operation to use when you are solving a problem? 2. How is the relationship between addition and subtraction like the relationship between multiplication and division? How is it different? 3. While working with fact families, you thought about decomposing numbers. 3a. What does it mean to decompose a number? 3b. How do fact families help you figure out the value for *N* in a sentence such as $N \div 2\frac{1}{2} = 1\frac{1}{4}$?

6-4: Covering and Surrounding
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Area and Perimeter Understand that perimeter is a measure of linear units needed to surround a two-dimensional shape and that area is a measure of square units needed to cover a two-dimensional shape

- Deepen the understanding of area and perimeter of rectangular and nonrectangular shapes
- Relate area to covering a figure
- Relate perimeter to surrounding a figure
- Analyze what it means to measure area and perimeter
- Develop and use formulas for calculating area and perimeter
- Develop techniques for estimating the area and perimeter of an irregular figure
- Explore relationships between perimeter and area, including that one can vary considerably while the other stays fixed
- Visually represent relationships between perimeter and area on a graph
- Solve problems involving area and perimeter of rectangles

Area and Perimeter of Parallelograms and Triangles Understand that the linear measurements of the base, height, and slanted height of parallelograms and triangles are essential to finding the area and perimeter of these shapes

- Analyze how the area of a triangle and the area of a parallelogram are related to each other and to the area of a rectangle
- Recognize that a triangle can be thought of as half of a rectangle whose sides are equal to the base and height of the triangle
- Recognize that a parallelogram can be decomposed into two triangles. Thus the area of a parallelogram is twice the area of a triangle with the same base and height as the parallelogram
- Know that the choice of base of a triangle (or parallelogram) is arbitrary but that the choice of the base determines the height
- Recognize that there are many triangles (or parallelograms) that can be drawn with the same base and height
- Develop formulas and strategies, stated in words or symbols, for finding the area and perimeter of triangles and parallelograms
- Find the side lengths and area of polygons on a coordinate grid
- Solve problems involving area and perimeter of parallelograms and triangles
- Solve problems involving area and perimeter of polygons by composing into rectangles or decomposing into triangles

Surface Area of Prisms and Pyramids and Volume of Rectangular Prisms Understand that the surface area of a three-dimensional shape is the sum of the areas of each two-dimensional surface of the shape and that the volume of a rectangular prism is a measure in cubic units of the capacity of the prism

- Extend the understanding of the volume of rectangular prisms
- Relate volume to filling a three-dimensional figure
- Extend understanding of the strategies for finding the volume of rectangular prisms to accommodate fractional side lengths
- Relate finding area of two-dimensional shapes to finding the surface area of three-dimensional objects
- Develop strategies for finding the surface area of three-dimensional objects made from rectangles and triangles
- Solve problems involving surface area of prisms and pyramids and volume of rectangular prisms

6-4 Covering and Surrounding: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Designing Bumper Cars: Extending and Building on Area and Perimeter	Investigation 2 Measuring Triangles	Investigation 3 Measuring Parallelograms	Investigation 4 Measuring Surface Area and Volume
Problem 1.1 Designing Bumper Car Rides: Area and Perimeter FQ: What are the formulas for finding the area and perimeter of a rectangle? Explain why they work. Problem 1.2 Building Storm Shelters: Constant Area, Changing Perimeter FQ: For a fixed area, what are the shape and perimeter of the rectangles with the greatest and least perimeters? Problem 1.3 Fencing in Spaces: Constant Perimeter, Changing Area FQ: For a fixed perimeter, what are the shape and area of the rectangles the greatest and least area?	Problem 2.1 Triangles on Grids: Finding Area and Perimeter of Triangles FQ: What is a formula for finding the area of a triangle? Problem 2.2 More Triangles: Identifying Base and Height FQ: Does it make any difference which side is used as the base when finding the area of a triangle? Problem 2.3 Making Families of Triangles: Maintaining the Base and the Height FQ: What can you say is true and what can you say is not true about triangles that have the same base and height? Problem 2.4 Designing Triangles Under Constraints FQ: What conditions for a triangle produce triangles that have the same area? Do they have the same shape? Explain.	Problem 3.1 Parallelograms and Triangles: Finding Area and Perimeter of Parallelograms FQ: What is a strategy for finding the area of a parallelogram? Explain why the strategy works. Problem 3.2 Making Families of Parallelograms: Maintaining the Base and the Height FQ: What can you say about two parallelograms that have the same base and height? Problem 3.3 Designing Parallelograms Under Constraints FQ: Under what conditions will two or more parallelograms have the same area? Do these parallelograms have the same shape? Explain. Problem 3.4 Polygons on Coordinate Grids FQ: How can you find the area of a polygon drawn on a coordinate graph? On grid paper?	Problem 4.1 Making Rectangular Boxes FQ: What is a strategy for finding the surface area of a rectangular prism? Explain why the strategy works. Problem 4.2 Filling the Boxes: Finding Volume FQ: What is a strategy for finding the volume of a rectangular prism? Explain why the strategy works. Problem 4.3 Designing Gift Boxes: Finding Surface Area FQ: What is a strategy for finding the surface area of three-dimensional object? Explain why the strategy works.
Mathematical Reflections 1a. Explain what area and perimeter of a figure means. 1b. Describe a strategy for finding the area and perimeter of any two-dimensional shape. 1c. Describe how you can find the area of a rectangle. Explain why this method works. 1d. Describe how you can find the perimeter of a rectangle. Explain why this method works. 2a. Consider all the rectangles with the same area. Describe the rectangle with the least perimeter. Describe the rectangle with the greatest perimeter. 2b. Consider all the rectangles with the same perimeter. Describe the rectangle with the least area. Describe the rectangle with the greatest area. 2c. Explain how graphing relationships between length and perimeter or length and area helps explain patterns between area and perimeter.	Mathematical Reflections 1a. Describe how to find the area of a triangle. Explain why your method works. 1b. Describe how to find the perimeter of a triangle. Explain why your method works. 2a. Does the choice of the base affect the area of a triangle? Does the choice of the base affect the perimeter of a triangle? Explain why or why not? 2b. What can you say about the area and perimeter of two triangles that have the same base and height? Give evidence to support your answer? 3. How is finding the area of a triangle related to finding the area of a rectangle? How is finding the perimeter of a triangle related to finding the perimeter of a rectangle?	Mathematical Reflections 1a. Describe how to find the area of a parallelogram. Explain why your method works. 1b. Describe how to find the perimeter of a parallelogram. Explain why your method works. 2a. Does the choice of the base change the area of a parallelogram? Does the choice of the base change the perimeter of a parallelogram? Explain why or why not? 2b. What can you say about the shape, area, and perimeter of two parallelograms that have the same base and height? Give evidence to support your answer? 3. How is the area of a parallelogram related to the area of a triangle and a rectangle? How is the perimeter of a parallelogram related to the perimeter of a triangle and a rectangle?	Mathematical Reflections 1a. What information do you need to find the volume of a rectangular prism? Describe a strategy to find the volume of a rectangular prism. 1b. What information do you need to find the surface area of a rectangular prism? Describe a strategy to find the surface area of a rectangular prism. 2a. Describe a strategy for finding the surface area of three-dimensional shapes made from rectangles and triangles. 2b. How does knowing the area of two-dimensional figures help you find the surface area of a three-dimensional shape?

6-5: Decimal Ops

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Numeric Estimation Understand that estimation can be used as a tool in a variety of situations, including as a way to check answers and make decisions

- Use estimates to solve problems and check answers

Decimal Operations Revisit and continue to develop meanings for the four arithmetic operations on rational numbers, and practice using algorithms to operate on decimals

- Recognize when addition, subtraction, multiplication, or division is the appropriate operation to solve a problem
- Use place value to develop understanding of algorithms and to relate operations with decimals to the same operations with fractions
- Extend understanding of multiplication and division of multidigit whole numbers
- Develop standard algorithms for multiplying and dividing decimals with the aid of, at most, paper and pencil
- Find a repeating or terminating decimal equivalent to a given fraction
- Solve problems using arithmetic operations on decimals, including finding unit rates

Variables and Number Sentences Use variables to represent unknown values and number sentences to represent relationships between values

- Write number sentences to represent relationships between both real-world and abstract values
- Use fact families to write and solve equivalent number sentences
- Use multiplication sentences to check division sentences

Percents Develop understanding of percents through various contexts, such as sales tax, tips, discounts, and percent increases

- Develop models for percent problems
- Write and solve number sentences involving percents

6-5 Decimal Ops: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Decimal Operations and Estimation	Investigation 2 Adding and Subtracting Decimals	Investigation 3 Multiplying and Dividing Decimals	Investigation 4 Using Percents
Problem 1.1 Out to Lunch: Matching Operations and Questions FQ: What signals in a real-world problem tell you which operation to use? Problem 1.2 Getting Close: Estimating Decimal Calculations FQ: When you work with decimal computations, what strategies can you use to estimate the results? Problem 1.3 Take a Hike: Connecting Ratios, Rates, and Decimals FQ: How can you express a unit rate as a decimal and use it to solve problems?	Problem 2.1 Getting Things in the Right Place: Adding Decimals FQ: What's the Difference? Subtracting Decimals Problem 2.2 What's the difference? Subtracting Decimals FQ: How do you subtract one decimal number from another? Problem 2.3 Connecting Operations: Fact Families FQ: Do fact families apply to operations with decimal numbers?	Problem 3.1 It's Decimal Times(s): Multiplying Decimals I FQ: How do you find the product of any two decimal numbers? Problem 3.2 It Works Every Time: Multiplying Decimals II FQ: What algorithm can be used to find any decimal product? Problem 3.3 How Many Times? Dividing Decimals I FQ: How can a decimal division problem be written in equivalent fraction and whole number form? Problem 3.4 Going the Long Way: Dividing Decimals II FQ: How can you carry out a decimal division using a method similar to long division of whole numbers? Problem 3.5 Challenging Cases: Dividing Decimals III FQ: How can you complete a long division problem that doesn't give a whole number quotient? That is, how do you express remainders in decimal form?	Problem 4.1 What's the Tax on This Item? FQ: How do you find the tax and the total cost of an item from a given selling price and tax rate? How do you find the base price from a given tax rate and amount? Problem 4.2 Computing Tips FQ: How do you find the tip and the total cost of a restaurant meal from a given meal price and tip rate? How do you find the meal price from a given tip percent and amount? Problem 4.3 Percent Discounts FQ: How do you find the discount and the total cost of an item from a given selling price and discount rate? How do you find the base price from a given discount rate and amount? How can you express a change in a given amount as a percent change? Problem 4.4 Putting Operations Together FQ: How do you decide which operations to perform when a problem involves decimals and percents?
Mathematical Reflections 1. How do you know when solving a problem that involves decimals requires addition? Subtraction? Multiplication? Division? 2. Describe a strategy that you use when estimating with decimals. Explain why it is helpful to you. 3. What is a unit rate? Describe how unit rates are useful.	Mathematical Reflections 1. How does interpreting decimals as fractions help you make sense of adding and subtracting decimals? Give an example to show your thinking. 2. How does the place-value interpretation of decimals help you add and subtract decimals? Give an example to show your thinking. 3. Describe algorithms for adding and subtracting any two decimal numbers.	Mathematical Reflections 1. What algorithm can be used to multiply any two decimal numbers? Explain why your algorithm works, and give an example that shows how it works. 2a. What algorithm can be used to divide any two decimal numbers? Explain why your algorithm works, and give an example that shows how it works. 2b. How can you predict whether a quotient will be a terminating decimal or a repeating decimal? 3a. What is the fact-family connection between decimal multiplication and division? 3b. How can you check the result of a division calculation? 3c. How can you check the result of a multiplication calculation?	Mathematical Reflections 1a. How do you find the tax on a purchase and calculate the final bill? Give an example, then write and solve a number sentence to illustrate your strategy. 1b. How do you find the price of a discounted item if you know the percent of the discount? Give an example, then write and solve a number sentence to illustrate your strategy. 1c. How do you find the cost of a purchase if you know the percent and the amount of the tax on the purchase? Give an example, then write and solve a number sentence to illustrate your strategy. 1d. How can you find the percent one number is of another? For example, what percent of 35 is 7? Write and solve a number sentence to illustrate your answer. 1e. How are all the number sentences in parts (a)-(d) the same? 2. How do you recognize when addition, subtraction, multiplication, and/or division of decimals is required to solve a problem?

6-6: Variables and Patterns
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Variables and Patterns (Relationships) Develop understanding of variables and how they are related

- Explore problem situations that involve variables and relationships
- Identify the dependent and independent variables and describe how they are related in a situation
- Interpret the "stories" told by patterns in tables and coordinate graphs of numeric (x, y) data
- Represent the pattern of change that relates two variables in words, data tables, graphs, and equations
- Investigate situations that change over time
- Examine increasing and decreasing patterns of change
- Compare linear and nonlinear patterns of change by using tables or graphs
- Use tables, graphs, and equations to find the value of a variable given the value of the associated variable
- Explore relationships that require graphing in all four quadrants
- Describe advantages and disadvantages of using words, tables, graphs, and equations to represent patterns of change relating two variables and make connections across those representations
- Write an equation to express the relationship between two variables in one and two operations: $y=mx$, $y=b+x$, and $y=b+mx$
- Calculate average speed and show how it is reflected in a table or graph and vice versa
- Recognize and express direct proportionality relationships with a unit rate ($y=mx$) and represent these relationships in rate tables and graphs
- Solve problems that involve variables

Expressions and Equations Develop understanding of expressions and equations

- Use properties of operations, including the Distributive Property and the Order of Operations, to write equivalent expressions for the dependent variable in terms of the independent variable
- Use tables, graphs, or properties of numbers such as the Distributive Property to show that two expressions are equivalent
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity
- Interpret and evaluate expressions in which letters stand for numbers and apply the Order of Operations as needed
- Recognize that equations are statements of equivalence between two expressions
- Solve linear equations of the forms $y=ax$, $y=b+x$, and $y=b+ax$ using numeric guess and check, tables of (x,y) values, and graphs or fact families
- Write an inequality and associate it with an equation to find solutions and graph the solutions on a number line

6-6 Variables and Patterns: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Variables, Tables and Graphs	Investigation 2 Analyzing Relationships among Variables	Investigation 3 Relating Variables with Equations	Investigation 4 Expressions, Equations, and Inequalities
Problem 1.1 Getting Ready to Ride: Data Tables and Graphs FQ: How can you construct a graph from a table of data that depicts change over time? How is the pattern of change represented in the graph? Problem 1.2 From Atlantic City to Lewes: Time, Rate, and Distance FQ: What are the advantages and disadvantages of tables and graphs in representing and describing the patterns of change in a variable over time? Problem 1.3 From Lewes to Chincoteague Island: Stories, Tables, and Graphs FQ: Which representation of data – table, graph, or written notes- seems to better show patterns of change in distance over time, and why? Problem 1.4 From Chincoteague to Colonial Williamsburg: Average Speed FQ: How do you calculate average speed for a trip? How do a table and graph of (time, distance) data show speed?	Problem 2.1 Renting Bicycles: Independent and Dependent Variables FQ: How do you analyze and compare the relationship between variables given in different representations? Problem 2.2 Finding Customers: Linear and Non-Linear Patterns FQ: How are the relationships between independent and dependent variables in this Problem different from those in Problem 2.1? How are the differences shown in tables and graphs of data? Problem 2.3 Predicting Profit: Four Quadrant Graphing FQ: How are the variables, *tour income* and *tour profit,* related to each other? How do you plot data points with one or both coordinates negative? Problem 2.4 What's the Story? Interpreting Graphs FQ: When the relationship between dependent and independent variables is displayed in a graph, what can you learn about the relationship from a rising graph, a level graph, and a falling graph?	Problem 3.1 Visit to Wild World: Equations with One Operation FQ: In what kinds of situations will the equation between dependent and independent variables be in the form $y = x + k$? $y = x – k$? $y = kx$? $y = x/k$? Problem 3.2 Moving, Texting, and Measuring: Using Rates and Rate Tables FQ: What can you tell about the relationship between dependent and independent variables in an equation of the form $y = mx$? How is that relationship shown in a table and a graph of sample (x, y) values? Why is the point (1, m) on every graph? Problem 3.3 Group Discounts and a Bonus Card: Equations with Two Operations FQ: How do you calculate values of y from an equation like $y = 3x + 5$ when values of x are given? How about $y = 5 + 3x$? When do you need such equations that involve two operations? Problem 3.4 Getting the Calculation Right: Expressions and Order of Operations FQ: When an equation relating two variables involves two or more operations, how do you use the equation to find values of the dependent variable from given values of the independent variable?	Problem 4.1 Taking the Plunge: Equivalent Expressions I FQ: Is it possible to have two different, but equivalent, expressions for a given situation? Explain. Problem 4.2 More Than One way to Say it: Equivalent Expressions II FQ: What does it mean to say that two algebraic expressions are equivalent? Problem 4.3 Putting it All Together: Equivalent Expressions III FQ: How can expressions such as $3x + 7x$ or $3(x + 2)$ be written in equivalent form? Problem 4.4 Finding the Unknown Value: Solving Equations FQ: What strategies can you use to solve equations in the forms $x + a = b$, $x – a = b$, $ax = b$, and $x \div a = b$ $(a \neq 0)$? Problem 4.5 It's Not Always Equal: Solving Inequalities FQ: How can you represent and find solutions for inequalities?
Mathematical Reflections 1. You can show patterns of change over time with tables, graphs, and written reports. 1a. What are the advantages and disadvantages of showing patterns with tables? 1b. What are the advantages and disadvantages of showing patterns with graphs? 1c. What are the advantages and disadvantages of showing patterns with written reports? 2a. How do you see patterns in the speed of a moving object by studying (time, distance) data in tables? 2b. How do you see patterns in the speed of a moving object by studying (time, distance) data in coordinate graphs?	Mathematical Reflections 1. The word variable is used often to describe conditions in science and business. 1a. Explain what the word variable means when it is used in situations like those you studied in this investigation. 1b. When are the words independent and dependent used to describe related variables? How are they used? 2. Suppose the values of a dependent variable increase as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following? 2a. a table of values for the two variables? 2b. a graph of values for the two variables? 3. Suppose the values of a dependent variable decrease as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following? 3a. a table of values for the two variables 3b. a graph of values for the two variables	Mathematical Reflections 1. What strategies help in finding equations to express relationships? 2 For relationships given by equations in the form $y = mx$: 2a. How does the value of y change as the value of x increases? 2b. How is the pattern of change shown in a table, graph, and equation of the function? 3a. In this unit, you have represented relationships between variables with tables, graphs, and equations. List some advantages and disadvantages of each of these representations. 3b. If the value of one variable in a relationship is known, describe how you can use a table, graph, or equation to find a value of the other variable.	Mathematical Reflections 1. What does it mean to say that two expressions are equivalent? How can you test the equivalence of two expressions? 2. What does it mean to *solve* an equation? What strategies are available for solving equations? 3. What does it mean to *solve* an inequality? What will graphs of such solutions look like for inequalities in the form $ax > b$ and $a + x < b$ (Assume *a* and *b* are both positive numbers). 4. Describe how expressions, equations, inequalities, and representations are used in this Unit. How are they related?

6-7: Data About Us
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Statistical Process Understand and use the process of statistical investigation
- Ask questions, collect and analyze data, and interpret data to answer questions
- Describe data with respect to its shape, center, and variability or spread
- Construct and use simple surveys as a method of collecting data

Attributes of Data Distinguish data and data types
- Recognize that data consist of counts or measurements of a variable, or an attribute; these observations comprise a distribution of data values
- Distinguish between categorical data and numerical data, and identify which graphs and statistics can be used to represent each kind of data

Multiple Representations for Displaying Data Display data with multiple representations
- Organize and represent data using tables, dot plots, line plots, ordered-value bar graphs, frequency bar graphs, histograms, and box-and-whisker plots
- Make informed decisions about which graphs or tables can be used to display a particular set of data
- Recognize that a graph shows the overall shape of a distribution, whether the data values are symmetrical around a central value, and whether the graph contains any unusual characteristics such as gaps, clusters, or outliers

Measures of Central Tendency and Variability Recognize that a single number may be used to characterize the center of a distribution of data and the degree of variability (or spread)
- Distinguish between and compute measures of central tendency (mean, median, and mode) and measures of spread (range, interquartile range (IQR), and mean absolute deviation (MAD))
- Identify how the median and mean respond to changes in the data values of a distribution
- Relate the choice of measures of central tendency and variability to the shape of the distribution and the context
- Describe the amount of variability in a distribution by noting whether the data values cluster in one or more areas or are fairly spread out
- Use measures of center and spread to compare data distributions

6-7 Data About Us: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 What's in a name? Organizing, Representing, and Describing Data	Investigation 2 Who's in Your Household? Using the Mean	Investigation 3 What's Your Favorite…? Measuring Variability	Investigation 4 What Numbers Describe Us? Using Graphs to Group Data
Problem 1.1 How Many Letters Are in a Name? FQ: What are "data"? How do you represent data using a frequency table or a line plot? How can you compare two distributions of data? Problem 1.2 Describing Name Lengths: What Are the Shape, Mode, and Range? FQ: What are the measures of central tendency and variability (or spread)? How do you compare and use mode and range? Problem 1.3 Describing Name Lengths: What is the Median? FQ: How do you identify and use the median? How can you compare two distributions of data using the medians?	Problem 2.1 What's a Mean Household Size? FQ: How do you go about finding a number that is a good estimate of typical household size based on the given data? Problem 2.2 Comparing Distributions With the Same Mean FQ: How do you interpret, compute, and use the mean? Problem 2.3 Making Choices: Mean or Median? FQ: How do the median and the mean respond to the data in a distribution? How do you choose which measure of center to use when describing what is typical? Problem 2.4 Who Else is in Your Household? Categorical and Numerical Data FQ: How do you distinguish different types of data? What statistics are used with different types of data?	Problem 3.1 Estimating Cereal Serving Sizes: Determining the IQR FQ: What information does the interquartile range provide about how data vary in a distribution? Problem 3.2 Connecting Cereal Shelf Location and Sugar Content: Describing Variability Using the IQR FQ: How is the interquartile range used to make comparisons among distributions? Problem 3.3 Is It Worth the Wait? Determining and Describing Variability Using the MAD FQ: What information does the mean absolute deviation provide about how data vary in a distribution?	Problem 4.1 Traveling to School: Histograms FQ: How can you use a histogram to help you interpret data? Problem 4.2 Jumping Rope: Box-and-Whisker Plots FQ: How can you interpret data using a box-and-whisker plot? Problem 4.3 How Much Taller Is a 6th Grader Than a 2nd Grader? Taking Variability Into Consideration FQ: How can you compare and contrast data represented by dot plots, histograms, and box plots?
Mathematical Reflections 1. The process of carrying out a statistical investigation involves asking a question, gathering and analyzing data, and interpreting the results to answer the question. Choose a data set from this Investigation. Use the data set to answer each question below. What was the question asked? How were the data collected? How were the data analyzed and represented? How did the results from the analysis help you answer the question? 2. You can represent a set of data using displays such as a data table, a frequency table, and a dot or line plot. Explain how these displays are related. 3. The median and mode are two measures of the center of a data distribution. The range is a measure of variability, or how spread out the data are. 3a. What does each measure of center tell you about the data set? 3b. Can the mode and the median for a data set have the same value? Can they have different values? Explain your answers. 3c. How does the range tell you how much the data vary? 3d. Suppose we add a new data value to the set of data. Does this new value affect the mode? The median? The range? Explain. 4. What strategies can you use to make comparisons among data sets	Mathematical Reflections 1. Describe a method for calculating the mean of a set of data. Explain why your method works. 2. You have used three measures of center – mode, median, and mean – to describe distributions. 2a. Why do you suppose they are called "measures of center"? 2b. What does each tell you about a set of data? 2c. How do you decide which measure of center to use when describing a distribution? 2d. Why might you want to include both the range and a measure of center when reporting a statistical summary? 3a. One student says you can only use the mode to describe categorical data, but you can use the mode, median, and mean to describe numerical data. Is the student correct? Explain. 3b. Can you find the range for categorical data? Explain.	Mathematical Reflections 1. Explain and illustrate the following words. 1a. Range 1b. Interquartile Range 1c. Mean absolute deviation 2a. Describe how you can use the range to compare how two data distributions vary. 2b. Describe how you can use the IQR to compare how two data distributions vary. 2c. Describe how you can use the MAD to compare how two data distributions vary.	Mathematical Reflections 1. Describe how you can display data using a histogram. 2. Describe how you can display data using a box plot. 3a. How can you use histograms to compare two data sets? 3b. How can you use box plots to compare two data sets? 4. Numerical data can be displayed using more than one type of graph. How do you decide when to use a dot plot, bar graph, histogram, or box plot?

7-1: Shapes and Designs
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Properties of Polygons Understand the properties of polygons that affect their shape

- Explore the ways that polygons are sorted into families according to the number and length of their sides and the size of their angles
- Explore the patterns among interior and exterior angles of a polygon
- Explore the patterns among side lengths in a polygon
- Investigate the symmetries of a shape—rotation or Reflections
- Determine which polygons fit together to cover a flat surface and why
- Reason about and solve problems involving various polygons

Relationships Among Angles Understand special relationships among angles

- Investigate techniques for estimating and measuring angles
- Use tools to sketch angles
- Reason about the properties of angles formed by parallel lines and transversals
- Use information about supplementary, complementary, vertical, and adjacent angles in a shape to solve for an unknown angle in a multi-step problem

Constructing Polygons Understand the properties needed to construct polygons

- Draw or sketch polygons with given conditions by using various tools and techniques such as freehand, use of a ruler and protractor, and use of technology
- Determine what conditions will produce a unique polygon, more than one polygon, or no polygon, particularly triangles and quadrilaterals
- Recognize the special properties of polygons, such as angle sum, side-length relationships, and symmetry, that make them useful in building, design, and nature
- Solve problems that involve properties of shapes

7-1 Shapes and Designs: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 The Family of Polygons	Investigation 2 Designing Polygons: The Angle Connection	Investigation 3 Designing Triangles and Quadrilaterals
Problem 1.1 Sorting and Sketching Polygons FQ: What properties do all polygons share? What properties do some sub-groups of polygons share? Problem 1.2 In a Spin: Angles and Rotations FQ: What are some common benchmark angles? What part of a full turn is each angle equal to? Problem 1.3 Estimating Measures of Rotations and Angles FQ: When a drawing shows two rays with a common endpoint, how many rotation angles are there? How would you estimate the measure of each angle? Problem 1.4 Measuring Angles FQ: How do you measure an angle with an angle ruler and a protractor? Problem 1.5 Design Challenge I: Drawing With Tools—Ruler and Protractor FQ: In a triangle, what measures of sides and angles give just enough information to draw a figure that is uniquely determined?	Problem 2.1 Angle Sums of Regular Polygons FQ: What is the size of each angle and the sum of all angles in a regular polygon with n sides? Problem 2.2 Angle Sums of Any Polygon FQ: What is the angle sum of any polygon with n sides? How do you know that your formula is correct? Problem 2.3 The Bees Do It: Polygons in Nature FQ: Which regular polygons can be used to tile a surface without overlaps or gaps, and how do you know that your answer is correct? Problem 2.4 The Ins and Outs of Polygons FQ: What is an exterior angle of a polygon, and what do you know about the measures of exterior angles?	Problem 3.1 Building Triangles FQ: What combinations of three side lengths can be used to make a triangle? How many different shapes are possible for such a combination of side lengths? Problem 3.2 Design Challenge II: Drawing Triangles FQ: What is the smallest number of side and angle measurements that will tell you how to draw an exact copy of any given triangle? Problem 3.3 Building Quadrilaterals FQ: What combinations of side lengths can be used to make a quadrilateral? How many different shapes are possible for any such combination of side lengths? Problem 3.4 Parallel Lines and Transversals FQ: When two parallel lines are cut by a transversal, what can be said about the eight angles that are formed? Problem 3.5 Design Challenge III: The Quadrilateral Game FQ: How are squares, rhombuses, rectangles, and trapezoids similar? How are they different?
Mathematical Reflections 1. What are the common properties of all polygons? 2. What does the measure in degrees tell you about an angle? What are some common benchmark angles? 3. What strategies can be used to estimate angle measures? To deduce angle measures from given information? To find accurate measurements with tools?	Mathematical Reflections 1. How is the number of sides related to the sum of the interior angles in a polygon? What about the sum of the exterior angles? 2. How is the measure of each interior angle related to the number of sides in a regular polygon? What about the measure of each exterior angle? 3. Which polygons can be used to tile a flat surface without overlaps or gaps? Why are those the only figures that work as tiles?	Mathematical Reflections 1. What information about combinations of angle sizes and side lengths provide enough information to copy a given triangle exactly? A quadrilateral? 2. Why are triangles so useful in building structures? What are the problems with quadrilaterals for building structures? 3. If two parallel lines are intersected by a transversal, which pairs of angles will have the same measure? 4. What does it mean to say a figure has symmetry? Provide examples with your explanation.

7-2: Accentuate the Negative
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Rational Numbers Develop an understanding that rational numbers consist of positive numbers, negative numbers, and zero
- Explore relationships between positive and negative numbers by modeling them on a number line
- Use appropriate notation to indicate positive and negative numbers
- Compare and order positive and negative rational numbers (integers, fractions, decimals, and zero) and locate them on a number line
- Recognize and use the relationship between a number and its opposite (additive inverse) to solve problems
- Relate direction and distance to the number line
- Use models and rational numbers to represent and solve problems

Operations With Rational Numbers Develop understanding of operations with rational numbers and their properties
- Develop and use different models (number line, chip model) for representing addition, subtraction, multiplication, and division
- Develop algorithms for adding, subtracting, multiplying, and dividing integers
- Recognize situations in which one or more operations of rational numbers are needed
- Interpret and write mathematical sentences to show relationships and solve problems
- Write and use related fact families for addition/subtraction and multiplication/division to solve simple equations
- Use parentheses and the Order of Operations in computations
- Understand and use the Commutative Property for addition and multiplication
- Apply the Distributive Property to simplify expressions and solve problems

7-2 Accentuate the Negative: Focus Questions (FQ) and Mathematical Reflections

<table>
<tr><th>Investigation 1
Extending the Number System</th><th>Investigation 2
Adding and Subtracting Rational Numbers</th><th>Investigation 3
Multiplying and Dividing Rational Numbers</th><th>Investigation 4
Properties of Operations</th></tr>
<tr><td>Problem 1.1
Playing Math Fever: Using Positive and Negative Numbers
FQ: How can you find the total value of a combination of positive and negative integers?

Problem 1.2
Extending the Number Line
FQ: How can you use a number line to compare two numbers?

Problem 1.3
From Sauna to Snowbank: Using a Number Line
FQ: How can you write a number sentence to represent a change on a number line, and how can you use a number line to represent a number sentence?

Problem 1.4
In the Chips: Using a Chip Model
FQ: How can you use a chip model to represent addition and subtraction?</td><td>Problem 2.1
Extending Addition to Rational Numbers
FQ: How can you predict whether the result of addition of two numbers will be positive, negative, or zero?

Problem 2.2
Extending Subtraction to Rational Numbers
FQ: How is a chip model or number line useful in determining an algorithm for subtraction?

Problem 2.3
The “+/-“ Connection
FQ: How are the algorithms for addition and subtraction of integers related?

Problem 2.4
Fact Families
FQ: What related sentence is equivalent to $4 + n = 43$ and makes it easier to find the value of n?</td><td>Problem 3.1
Multiplication Patterns With Integers
FQ: How is multiplication of two integers represented on a number line and chip board?

Problem 3.2
Multiplication of Rational Integers
FQ: What algorithm can you use for multiplying integers?

Problem 3.3
Division of Rational Numbers
FQ: What algorithm can you use for dividing integers? How are multiplication and division related?

Problem 3.4
Playing the Integer Product Game: Applying Multiplication and Division of Integers
FQ: What patterns do you notice on the game board for the Integer Product Game that can help you?</td><td>Problem 4.1
Order of Operations
FQ: Does the Order of Operations work for integers? Explain.

Problem 4.2
The Distributive Property
FQ: How can you use the Distributive Property to expand an expression or factor an expression that involves integers?

Problem 4.3
What Operations are Needed?
FQ: What information in a problem is useful to help you decide which operation to use to solve the problem?</td></tr>
<tr><td>Mathematical Reflections

1. How do decide which of two numbers is greater when
1a. both numbers are positive?
1b. both numbers are negative?
1c. one number is positive and one number is negative?

2. How does a number line help you compare numbers?

3. When you add a positive number and a negative number, how do you determine the sign of the answer?

4. If you are doing a subtraction problem on a chip board, and the board does not have enough chips of the color you wish to subtract, what can you do to make the subtraction possible?</td><td>Mathematical Reflections

1a. What algorithm(s) will produce the correct result for the sum “$a + b$,” where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works.

1b. What algorithm(s) will produce the correct result for the difference “$a – b$,” where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works.

2. How can any difference “$a – b$” be restated as an equivalent addition statement, where a and b each represent any rational number?

3a. What does it mean to say that an operation is commutative?

3b. Describe some ways that the additive inverse of a number is important.</td><td>Mathematical Reflections

1. Give an example of a multiplication problem, involving two integers, in which the product is
1a. less than 0.
1b. greater than 0.
1c. equal to 0.
1d. In general, describe the signs of the factors for each product in parts (a)–(c).

2. Give an example of a division problem, involving two integers, in which the quotient is
2a. less than 0.
2b. Greater than 0.
2c. Equal to 0.
2d. In general, describe the signs of the dividend and divisor for each quotient in parts (a)–(c).

3a. Suppose three numbers are related by an equation of the form $a \cdot b = c$, where a, b, and c are not equal to 0. Write two related number sentences using multiplication.

3b. Suppose three numbers are related by an equation of the form $a \div b = c$, where a, b, and c are not equal to 0. Write two related number sentences using multiplication.

4. Which operations on integers are commutative? Give numerical examples to support your answer.</td><td>Mathematical Reflections

1a. What is the Order of Operations? Why is the Order of Operations important?

1b. Give an example of a numerical expression in which the use of parentheses changes the result of the computation.

2. Describe how the Distributive Property relates addition and multiplication. Give numerical examples.</td></tr>
</table>

7-3: Stretching and Shrinking
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Similar Figures Understand what it means for figures to be similar

- Identify similar figures by comparing corresponding sides and angles
- Use scale factors and ratios to describe relationships among the side lengths, perimeters, and areas of similar figures
- Generalize properties of similar figures
- Recognize the role multiplication plays in similarity relationships
- Recognize the relationship between scale factor and ratio in similar figures
- Use informal methods, scale factors, and geometric tools to construct similar figures (scale drawings)
- Compare similar figures with nonsimilar figures
- Distinguish algebraic rules that produce similar figures from those that produce nonsimilar figures
- Use algebraic rules to produce similar figures
- Recognize when a rule shrinks or enlarges a figure
- Explore the effect on the image of a figure if a number is added to the x- or y-coordinates of the figure's vertices

Reasoning with Similar Figures Develop strategies for using similar figures to solve problems

- Use the properties of similarity to find distances and heights that cannot be measured directly
- Predict the ways that stretching or shrinking a figure will affect side lengths, angle measures, perimeters, and areas
- Use scale factors or ratios to find missing side lengths in a pair of similar figures
- Use similarity to solve real-world problems

7-3 Stretching and Shrinking: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Enlarging and Reducing Shapes	Investigation 2 Similar Figures	Investigation 3 Scaling Perimeter and Area	Investigation 4 Similarity and Ratios
Problem 1.1 Solving a Mystery: An Introduction to Similarity FQ: What does it mean for two figures to be similar? Problem 1.2 Scaling Up and Down: Corresponding Sides and Angles FQ: When you copy a figure at a certain scale factor (e.g. 150%), how does this value affect the measurements of the new figure?	Problem 2.1 Drawing Wumps: Making Similar Figures FQ: How can you determine if two shapes are similar by looking at the rule for producing specific coordinates for the image? Problem 2.2 Hats Off to the Wumps: Changing a Figure's Size and Location FQ: What types of coordinate rules produce similar figures? Nonsimilar figures? For a pair of similar figures, how can you use a coordinate rule to predict the side lengths of the image? Problem 2.3 Mouthing Off and Nosing Around: Scale Factors FQ: How can you decide whether or not two shapes are similar?	Problem 3.1 Rep-Tile Quadrilaterals: Forming Rep-Tiles With Similar Quadrilaterals FQ: What types of quadrilaterals are rep-tiles? How do rep-tiles show that the scale factors and areas of similar quadrilaterals are related? Problem 3.2 Rep-Tile Triangles: Forming Rep-Tiles With Similar Figures FQ: Which types of triangles are rep-tiles? Explain. Problem 3.3 Designing Under Constraints: Scale Factors and Similar Shapes FQ: How can you use scale factors to draw similar figures or to find missing side lengths in similar figures? Problem 3.4 Out of Reach: Finding Lengths with Similar Triangles FQ: How can you use similar triangles to find a distance that is difficult to measure directly?	Problem 4.1 Ratios Within Similar Parallelograms FQ: What information does the ratio of adjacent side lengths within a rectangle give you? Problem 4.2 Ratios Within Similar Triangles FQ: For a pair of triangles, if the measures of corresponding angles are equal, how can you use ratios of side lengths to determine whether or not the triangles are similar? Problem 4.3 Finding Missing Parts: Using Similarity to Find Measurements FQ: If two shapes are similar, how can you use information about the shapes to find unknown side lengths, perimeters, and areas? Problem 4.4 Using Shadows to Find Heights: Using Similar Triangles FQ: How can you use similar triangles to estimate the heights of tall objects?
Mathematical Reflections 1a. When you enlarge or reduce a figure, what features stay the same? 1b. When you enlarge or reduce a figure, what features change? 2. Rubber-band stretchers, copy machines, and projectors all make images that are similar to the original shapes. What does it mean for two shapes to be similar? Complete the sentence below: *"Two geometric shapes are similar when…"*	Mathematical Reflections 1. If two shapes are similar, what is the same about them and what is different? 2a. What does the scale factor tell you about two similar figures? 2b. How does the coordinate rule for making two similar shapes relate to the scale factor? 3. Rubber band stretchers, copy machines, and coordinate grids all made images that are similar to (or scale drawings of) the original shapes. What does it mean to say two shapes are similar? Build on your statement from Mathematical Reflections 1: *"Two geometric shapes are similar when…"*	Mathematical Reflections 1a. If two polygons are similar, how can find the scale factor from one polygon to the other? Give specific examples. 1b. Suppose you are given a polygon. How can you draw a similar figure? 2. What does the scale factor between two similar figures tell you about the 2a. side lengths? 2b. perimeters? 2c. areas? 2d. angles? 3. If two figures are similar, how can you find a missing side length? 4. Describe how you can find the measure of a distance that you cannot measure directly. 5. What does it mean to say two shapes are similar? After completing Investigation 3, how can you build on your statements from Mathematical Reflections 1 and 2? *"Two geometric shapes are similar when…"*	Mathematical Reflections 1. If two triangles, rectangles, or parallelograms are similar, 1a. How does the ratio of two side lengths within one figure compare to the ratio of the corresponding side lengths in the other figure? 1b. What does the scale factor from one figure to the other tell you about the figures? 2a. Describe at least two ways to find a missing side length in a pair of similar figures. 2b. How can you find the height of an object that cannot be measured directly? 3. What does it mean to say that two shapes are similar? After exploring with ratios, build on your statements from Mathematical Reflections 1, 3, and 3: *"Two geometric shapes are similar when…*

7-4: Comparing and Scaling
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Ratios, Rates, and Percents Understand ratios, rates, and percents

- Use ratios, rates, fractions, differences, and percents to write statements comparing two quantities in a given situation
- Distinguish between and use both part-to-part and part-to-whole ratios in comparisons
- Use percents to express ratios and proportions
- Recognize that a rate is a special ratio that compares two measurements with different units
- Analyze comparison statements made about quantitative data for correctness and quality
- Make judgments about which kind of comparison statements are most informative or best reflect a particular point of view in a specific situation

Proportionality Understand proportionality in tables, graphs, and equations

- Recognize that constant growth in a table, graph, or equation is related to proportional situations
- Write an equation to represent the pattern in a table or graph of proportionally related variables
- Relate the unit rate and constant of proportionality to an equation, graph, or table describing a proportional situation

Reasoning Proportionally Develop and use strategies for solving problems that require proportional reasoning

- Recognize situations in which proportional reasoning is appropriate to solve the problem
- Scale a ratio, rate, percent, or fraction to make a comparison or find an equivalent representation
- Use various strategies to solve for an unknown in a proportion, including scaling, rate tables, percent bars, unit rates, and equivalent ratios
- Set up and solve proportions that arise from real-world applications, such as finding discounts and markups and converting measurement units

7-4 Comparing and Scaling: Focus Questions (FQ) and Mathematical Reflections

<table>
<tr><th>Investigation 1
Ways of Comparing: Ratios and Proportions</th><th>Investigation 2
Comparing and Scaling Rates</th><th>Investigation 3
Markups, Markdowns, and Measures: Using Ratios, Percents, and Proportions</th></tr>
<tr><td>Problem 1.1
Surveying Opinions: Analyzing Comparison Statements
FQ: What do different comparisons of quantities tell you about their relationship?

Problem 1.2
Mixing Juice: Comparing Ratios
FQ: What strategies do you use to determine which mix is the most orangey?

Problem 1.3
Time to Concentrate: Scaling Ratios
FQ: When you scale up a recipe and change the units, like from cups to ounces, what are some of the issues you have to deal with?

Problem 1.4
Keeping Things in Proportion: Scaling to Solve Proportions
FQ: What strategies can you use to find a missing value in a proportion? What is your preferred strategy and why?</td><td>Problem 2.1
Sharing Pizza: Comparison Strategies
FQ: How can you determine whether two ratios are equivalent or find which of two ratios is more favorable?

Problem 2.2
Comparing Pizza Prices: Scaling Rates
FQ: How can you use rate tables to find missing values? How are rate tables similar to scaling quantities and solving proportions?

Problem 2.3
Finding Costs: Unit Rate and Constant of Proportionality
FQ: How can you find a unit rate in a description, an equation, a table, or a graph?</td><td>Problem 3.1
Commissions, Markups, and Discounts: Proportions With Percents
FQ: How can you use proportions and percent tables to find various percentages of a value when you know a certain percentage of the same value?

Problem 3.2
Measuring to the Unit: Measurement Conversions
FQ: How can you use unit rates, proportions, equations, and rate tables to scale a variety of units?

Problem 3.3
Mixing it Up: Connecting Ratios, Rates, Percents, and Proportions
FQ: How can you use scale factors, rate tables, proportions, equations, or graphs to find amounts of a mixture, given the proportions?</td></tr>
<tr><td>Mathematical Reflections

1a. In this Investigation you have used ratios, percents, fractions, and differences to make comparison statements. How have you found these ideas helpful?
1b. Give examples to explain how part-to-part ratios are different from, but related to, part-to-whole ratios.

2. How can you use scaling or equivalent rations
2a. to solve a proportion? Give an example.
2b. To make a decision? Give an example.

3. You learned about scaling in Stretching and Shrinking. You learned about proportions and rates in Comparing and Scaling. How are the ideas in these two Units the same? How are they different?

4. Describe the connections you have found among unit rates, proportions, and rate tables.</td><td>Mathematical Reflections

1a. How are tables, graphs, and equations helpful when you work with proportions?

1b. How can you identify a unit rate or constant of proportionality in a table? In a graph? In an equation?

2. How are unit rates useful?

3. How is finding a unit rate similar to solving a proportion?</td><td>Mathematical Reflections

1. What strategies have you learned for solving proportions?

2. Describe a strategy for converting a rate measured in one pair of units to a rate measured in a different pair of units. For example, how would you convert ounces per cup to pounds per gallon?

3. You learned about scaling in Stretching and Shrinking. You learned about proportions and rates in Comparing and Scaling. How are the ideas in these two Units the same? How are they different?

4. Describe the connections you have found among unit rates, proportions, and rate tables.</td></tr>
</table>

7-5: Moving Straight Ahead
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Linear Relationships Recognize problem situations in which two variables have a linear relationship

- Identify and describe the patterns of change between the independent and dependent variables for linear relationships represented by tables, graphs, equations, or contextual settings
- Construct tables, graphs, and symbolic equations that represent linear relationships
- Identify the rate of change between two variables and the x- and y-intercepts from graphs, tables, and equations that represent linear relationships
- Translate information about linear relationships given in a contextual setting, a table, a graph, or an equation to one of the other forms
- Write equations that represent linear relationships given specific pieces of information, and describe what information the variables and numbers represent
- Make a connection between slope as a ratio of vertical distance to horizontal distance between two points on a line and the rate of change between two variables that have a linear relationship
- Recognize that $y=mx$ represents a proportional relationship
- Solve problems and make decisions about linear relationships using information given in tables, graphs, and equations

Equivalence Understand that the equality sign indicates that two expressions are equivalent

- Recognize that the equation $y=mx+b$ represents a linear relationship and means that $mx+b$ is an expression equivalent to y
- Recognize that linear equations in one unknown, $k=mx+b$ or $y=m(t)+b$, where k, t, m, and b are constant numbers, are special cases of the equation $y=mx+b$
- Recognize that finding the missing value of one of the variables in a linear relationship, $y=mx+b$, is the same as finding a missing coordinate of a point (x,y) that lies on the graph of the relationship
- Solve linear equations in one variable using symbolic methods, tables, and graphs
- Recognize that a linear inequality in one unknown is associated with a linear equation
- Solve linear inequalities using graphs or symbolic reasoning
- Show that two expressions are equivalent
- Write and interpret equivalent expressions

7-5 Moving Straight Ahead: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Walking Rates	Investigation 2 Exploring Linear Relationships with Graphs and Tables	Investigation 3 Solving Equations	Investigation 4 Exploring Slope: Connecting Rates and Ratios
Problem 1.1 Walking Marathons: Finding and Using Rates FQ: What equation represents the relationship between the time and the distance you walk at a constant rate? What are the dependent and independent variables? Problem 1.2 Walking Rates and Linear Relationships: Tables, Graphs, and Equations FQ: How can you predict whether a relationship is linear from a table, a graph, or an equation that represents the relationship? Problem 1.3 Raising Money: Using Linear Relationships FQ: What is the pattern of change in a linear relationship? Problem 1.4 Using the Walkathon Money: Recognizing Linear Relationships FQ: How can you determine if a linear relationship is increasing or decreasing?	Problem 2.1 Henri and Emile's Race: Finding the Point of Intersection FQ: When is it helpful to use a graph or table to solve a problem? Problem 2.2 Crossing the Line: Using Tables, Graphs, and Equations FQ: How does the pattern of change for a linear relationship appear in a table, a graph, or an equation? Problem 2.3 Comparing Costs: Comparing Relationships FQ: How can you decide if a table or an equation represents a linear relationship? Problem 2.4 Connecting Tables, Graphs, and Equations FQ: How are solutions of an equation of the form y = b + mx related to the graph and the table for the same relationship?	Problem 3.1 Solving Equations Using Tables and Graphs FQ: How are the coordinates of a point on a line or in a table related to the equation of the line? Problem 3.2 Mystery Pouches in the Kingdom of Montarek: Exploring Equality FQ: What does equality mean? Problem 3.3 From Pouches to Variables: Writing Equations FQ: How can the properties of equality be used to solve linear equations? Problem 3.4 Solving Linear Equations FQ: What are some strategies for solving linear equations? Problem 3.5 Finding the Point of Intersection: Equations and Inequalities FQ: How can you find when two expressions are equal, or when one expression is greater or less than the other?	Problem 4.1 Climbing Stairs: Using Rise and Run FQ: How is the steepness of a set of stairs related to a straight-line graph? Problem 4.2 Finding the Slope of a Line FQ: How can you find the y-intercept and the slope of a line from data in a table, graph, or equation? Problem 4.3 Exploring Patterns with Lines FQ: How can you predict if two lines are parallel or perpendicular from their equations? Problem 4.4 Pulling it All Together: Writing Equations for Linear Relationships FQ: What information do you need to write an equation for a linear relationship? Is the expression for the dependent variable always the same?
Mathematical Reflections 1. Describe how the dependent variable changes as the independent variable changes in a linear relationship. Give examples. 2. How does the pattern of change between two variables in a linear relationship show up in 2a. a contextual situation? 2b. a table? 2c. a graph? 2d. an equation?	Mathematical Reflections 1a. Explain how the information about a linear relationship is represented in a table, a graph, or an equation. 1b. Describe several real-world situations that can be modeled by equations of the form y = mx + b and y = mx. Explain how the latter equation represents a proportional relationship. 2a. Explain how a table or graph that represent a linear relationship can be used to solve a problem. 2b. Explain how you have used an equation that represents a linear relationship to solve a problem.	Mathematical Reflections 1a. Suppose that, in an equation with two variables, you know the value of one of the variables. Describe a method for finding the value of the other variable using the properties of equality. Give an example to illustrate your method. 1b. Compare the method you described in part (a) to the methods of using a table or a graph to solve linear equations. 2a. Explain how an inequality can be solved by methods similar to those used to solve linear equations. 2b. Describe a method for finding the solution to an inequality using graphs. 3. Give an example of two equivalent expressions that were used in this investigation. Explain why they are equivalent.	Mathematical Reflections 1. Explain what the slope of a line is. How does finding the slope compare to finding the rate of change between two variables in a linear relationship? 2. How can you find the slope of a line from 2a. an equation? 2b. a graph? 2c. a table of values of the line? 2d. the coordinates of two points on the line? 3. For parts (a) and (b), explain how you can write an equation of a line from the information. Use examples to illustrate your thinking. 3a. the slope and the y-intercept of the line 3b. two points on the line

7-6 What Do You Expect
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Experimental and Theoretical Probabilities Understand experimental and theoretical probabilities

- Recognize that probabilities are useful for predicting what will happen over the long run
- For an event described in everyday language, identify the outcomes in a sample space that compose the event
- Interpret experimental and theoretical probabilities and the relationship between them and recognize that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers
- Distinguish between outcomes that are equally likely or not equally likely by collecting data and analyzing experimental probabilities
- Realize that the probability of simple events is a ratio of favorable outcomes to all outcomes in the sample space
- Recognize that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
- Determine the fairness of a game

Reasoning With Probability Explore and develop probability models by identifying possible outcomes and analyze probabilities to solve problems

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process
- Represent sample spaces for simple and compound events and find probabilities using organized lists, tables, tree diagrams, area models, and simulation
- Realize that, just as with simple events, the probability of a compound event is a ratio of favorable outcomes to all outcomes in the sample space
- Design and use a simulation to generate frequencies for simple and compound events
- Analyze situations that involve two or more stages (or actions) called *compound events*
- Use area models to analyze the theoretical probabilities for two-stage outcomes
- Analyze situations that involve binomial outcomes
- Use probability to calculate the long-term average of a game of chance
- Determine the expected value of a probability situation
- Use probability and expected value to make a decision

7-6 What Do You Expect: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 A First Look at Chance	Investigation 2 Experimental and Theoretical Probability	Investigation 3 Making Decisions With Probability	Investigation 4 Analyzing Compound Events Using an Area Model	Investigation 5 Binomial Outcomes
Problem 1.1 Choosing Cereal: Tossing a Coin to Find Probabilities FQ: How does collecting more data help you predict the outcome of a situation? Problem 1.2 Tossing Paper Cups: Finding More Probabilities FQ: How does modeling with an experiment help you determine possible outcomes and the likelihood of each outcome? Problem 1.3 One More Try: Finding Experimental Probabilities FQ: How do you determine the relative frequency of an outcome? Problem 1.4 Analyzing Events: Understanding Equally Likely FQ: How can you determine whether the outcomes of a probability event are all equally likely, and why would this information matter?	Problem 2.1 Predicting to Win: Finding Theoretical Probabilities FQ: How does experimental probability compare to theoretical probability for a given situation? Problem 2.2 Choosing Marbles: Developing Probability Models FQ: What are some properties of theoretical probabilities? Problem 2.3 Designing a Fair Game: Pondering Possible and Probable FQ: How can you decide whether a game is fair or not? Problem 2.4 Winning the Bonus Prize: Using Strategies to Find Theoretical Probabilities FQ: How can you determine all of the probabilities for a compound event?	Problem 3.1 Designing a Spinner to Find Probabilities FQ: How do you determine probability using a spinner? Problem 3.2 Making Decisions: Analyzing Fairness FQ: When using a tool to simulate a fair game, what things must you consider? Problem 3.3 Roller Derby: Analyzing a Game FQ: How does understanding probability help you design a winning strategy? Problem 3.4 Scratching Spots: Designing and Using a Simulation FQ: How can you design a simulation to determine probability?	Problem 4.1 Drawing Area Models to Find the Sample Space FQ: How can an area model represent a situation to help analyze probabilities? Problem 4.2 Making Purple: Area Models and Probability FQ: How can you use experimental or theoretical probabilities of a compound event to predict the number of times one particular combination will occur out of any given number of repetitions of the event? Problem 4.3 One-and-One Free Throws: Simulating a Probability Situation FQ: How is an area model for the one-and-one free-throw situation like or unlike the area model for the Making Purple game? Problem 4.4 Finding Expected Value FQ: How is expected value different from probabilities of outcomes?	Problem 5.1 Guessing Answers: Finding More Expected Values FQ: If you do not know the answers to a true/false test, what is the probability that you can get a good score with random guesses? Problem 5.2 Ortonville Binomial Probability FQ: What patterns are there in models for binomial probability situations that are equally likely? How do these patterns help you answer probability questions? Problem 5.3 A Baseball Series: Expanding Binomial Probability FQ: If two teams are evenly matched, how do binomial probabilities help you figure out the probabilities that a winner of the required number of games will occur after a certain number of games?
Mathematical Reflections 1. How do you find the experimental probability that a particular result will occur? Why is it called the experimental probability? 2. In an experiment, are 30 trials as good as 500 trials to predict the chances of a result? Explain/ 3. What does it mean for results to be equally likely?	Mathematical Reflections 1. Describe how you can find the theoretical probability of an outcome. Why is it called theoretical probability? 2a. Suppose two people do an experiment to estimate the probability of an outcome. Will they get the same probabilities? Explain. 2b. Two people analyze a situation to find the theoretical probability of an outcome. Will they get the same probabilities? Explain. 2c. One person uses an experiment to estimate the probability of an outcome. Another person analyzes the situation to find the theoretical probability of the outcome. Will they get the same probabilities? Explain. 3. What does it mean for a game to be fair? 4. What is a sample space, and how can it be represented?	Mathematical Reflections 1. Describe a situation in which you and a friend can use probability to make a decision. Can the probabilities of the outcomes be determined both experimentally and theoretically? Why or why not? 2. Describe a situation in which it is difficult or impossible to find the theoretical probabilities of the outcomes. 3. Explain what it means of a probability situation to be fair. 4. Describe some of the strategies for determining the theoretical probabilities for situations in this unit. Give an example of a situation for each of the strategies.	Mathematical Reflections 1. Describe four probability situations that involve two actions. Describe the outcomes for these situations. 2. You can use an area model or a simulation to determine the probability of a situation that involves two actions. Explain how each of these is used. 3. Describe how you would calculate the expected value for a probability situation. 4. Expected value is sometimes called the longer-term average. Explain why this makes sense.	Mathematical Reflections 1. Describe five different binomial situations. Explain why they are binomial situations. 2. Tossing a coin three times is an example of a situation involving a series of three actions, each with two equally likely outcomes. 2a. Pick one of the situation in Question 1. Describe a series of three actions, each with two equally likely outcomes. Make a list of all the possible outcomes. 2b. Write a question about your situation that can be answered by your list. 3. As you increase the number of actions for a binomial situation, what happens to the total number of possible outcomes? For example, suppose you increase the number of times a coin is tossed. What happens to the total number of outcomes?

7-7: Filling and Wrapping

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Surface Areas and Volumes of Polygonal Prisms and Cylinders Understand surface areas and volumes of prisms and cylinders and how they are related

- Describe prisms by using their vertices, faces, and edges
- Visualize three-dimensional shapes and the effects of slicing those shapes by planes
- Deepen understanding of volumes and surface areas of rectangular prisms
- Estimate and calculate surface areas and volumes of polygonal prisms by relating them to rectangular prisms
- Explore the relationships between the surface areas and volumes of prisms
- Relate surface areas and volumes for common figures, especially optimization of surface area for fixed volume
- Predict the effects of scaling dimensions on linear, surface area, and volume measures of prisms, cylinders, and other figures
- Investigate the relationship between volumes of prisms and volumes of cylinders as well as the relationship between surface areas of prisms and surface areas of cylinders
- Use volumes and surface areas of prisms to develop formulas for volumes and surface areas of cylinders
- Discover that volumes of prisms and cylinders can be calculated as the product of the area of the base and the height
- Solve problems involving surface areas and volumes of solid figures

Areas and Circumferences of Circles Understand the areas and circumferences of circles and how they are related

- Relate area of a circle to covering a figure and circumference to surrounding a figure
- Estimate and calculate areas and circumferences of circles
- Explore the relationship between circle radius (or diameter) and circumference
- Explore the relationship between circle radius (or diameter) and area
- Investigate the connection of π to area calculation by estimating the number of radius squares needed to cover a circle
- Investigate the relationship between area and circumference of a circle
- Solve problems involving areas and circumferences of circles

Volumes of Spheres and Cones Understand the relationships between the volumes of cylinders and the volumes of cones and spheres

- Relate volumes of cylinders to volumes of cones and spheres
- Estimate and calculate volumes of spheres and cones
- Solve problems involving surface areas and volumes of spheres and cones

7-7 Filling and Wrapping: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Building Smart Boxes: Rectangular Prisms	Investigation 2 Polygonal Prisms	Investigation 3 Area and Circumference of Circles	Investigation 4 Cylinders, Cones, and Spheres
Problem 1.1 How Big Are Those Boxes? Finding Volume FQ: How do you calculate the surface area and volume of a rectangular prism? Problem 1.2 Optimal Containers I: Finding Surface Area FQ: Suppose you design a box in the shape of a rectangular prism with a volume of 24 cm^3. What are the shape and dimensions of the box that has minimum surface area? Problem 1.3 Optimal Containers II: Finding the Least Surface Area FQ: What are the dimensions of the rectangular prism that has the least surface area for a given volume? Problem 1.4 Compost Containers: Scaling Up Prisms FQ: As you change the dimensions of a rectangular prism by a certain scale factor, how do the surface area and volume of the prism change?	Problem 2.1 Folding Paper: Surface Area and Volume of Prisms FQ: For a prism with fixed height and fixed lateral area, how do the volume and surface area of the prism change as the number of sides increases? Problem 2.2 Packing a Prism: Calculating Volume of Prisms FQ: What general strategy can be used to find the volume of any prism—triangular, rectangular, pentagonal, and so on? Problem 2.3 Slicing Prisms and Pyramids FQ: What surface shapes and three-dimensional figures can be created by slicing a rectangular prism in various directions?	Problem 3.1 Going Around in Circles: Circumference FQ: What is the relationship between the diameter or radius of a circle and its circumference? Problem 3.2 Pricing Pizza: Connecting Area, Diameter, and Radius FQ: How does the area of a circle increase as the circle's radius and diameter increase? Problem 3.3 Squaring a Circle to Find is Area FQ: What is the relationship between the area of a circle and its radius? Problem 3.4 Connecting Circumference and Area FQ: What is the relationship between the circumference and area of a circle?	Problem 4.1 Networking: Surface Area of Cylinders FQ: How can you calculate the surface area of a cylinder? Why does that strategy work? Problem 4.2 Wrapping Paper: Volume of Cylinders FQ: How can you calculate the volume of a cylinder? How is the procedure similar to calculating the volume of a prism? Problem 4.3 Comparing Juice Containers: Comparing Surface Areas FQ: How does the surface area of a cylinder compare to the surface area of a rectangular prism for a given volume? Problem 4.4 Filling Cones and Spheres FQ: If a sphere and a cone have the same dimensions as a cylinder, how do the volumes compare? What formulas for volume of a sphere and the volume of a cone can you write using these relationships? Problem 4.5 Comparing Volumes of Spheres, Cylinders, and Cones FQ: What are some relationships you can use involving a cone, a sphere, and a cylinder with the same dimensions?
Mathematical Reflections 1. How can you calculate the volume and surface area of a rectangular prism from measures of its length, width, and height? Explain why this works. 2. How are the surface area and volume of a rectangular prism related to each other? 3. How will the surface area and volume of a prism change in each of the following cases? 3a. You increase or reduce one dimension by a scale factor of *f*. 3b. You increase or reduce two dimensions by a scale factor of *f*. 3c. You increase or reduce all three dimensions by a scale factor of *f*.	Mathematical Reflections 1. How can you find the surface area of any right prism? Explain why your method works. 2. How can you find the volume of any right prism? Explain why your method works 3. What two- and three-dimensional shapes result when a right rectangular prism is cut by 3a. a horizontal slice? 3b. a vertical slice? 3c. a slanted slice?	Mathematical Reflections 1. How can you find the circumference and area of a circle from measures of its radius or diameter? 2. How is the challenge of finding circumferences and areas of circles similar to that of finding perimeters and areas of polygons such as triangles, rectangles, and other parallelograms? In what ways are those tasks different?	Mathematical Reflections 1a. Compare the task of finding the circumference of the base and the surface area of a cylinder to that of finding the perimeter of the base and the surface area a prism. 1b. Compare the task of finding the volume of cylinders to that of finding the volume of prisms. 1c. How can you find the circumference of the base, the surface area, and the volume of a cylinder from measures of its radius or diameter and its height? Explain why your formulas make sense. 1d. How do the surface area and the volume of a cylinder change if both the radius and height are changed by a factor of *f*? 2a. How is the task of finding the volumes of spheres and cones similar to that of finding the volumes of prisms and cylinders? In what ways are those tasks different? 2b. How can you find the volume of a sphere or a cone from measures of its dimensions?

7-8: Samples and Populations
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

The Process of Statistical Investigation Deepen the understanding of the process of statistical investigation and apply this understanding to samples

- Pose questions, collect data, analyze data, and interpret data to answer questions

Analysis of Samples Understand that data values in a sample vary and that summary statistics of samples, even same-sized samples, taken from the same population also vary

- Choose appropriate measures of center (mean, median, or mode) and spread (range, IQR, or MAD) to summarize a sample
- Choose appropriate representations to display distributions of samples
- Compare summary statistics of multiple samples drawn from either the same population or from two different populations and explain how the samples vary

Design and Use of Simulations Understand that simulations can model real-world situations

- Design a model that relies on probability concepts to obtain a desired result
- Use the randomly generated frequencies for events to draw conclusions

Predictions and Conclusions About Populations Understand that summary statistics of a representative sample can be used to gain information about a population

- Describe the benefits and drawbacks to various sampling plans
- Use random-sampling techniques to select representative samples
- Apply concepts from probability to select random samples from populations
- Explain how sample size influences the reliability of sample statistics and resulting conclusions and predictions
- Explain how different sampling plans influence the reliability of sample statistics and resulting conclusions and predictions
- Use statistics from representative samples to draw conclusions about populations
- Use measures of center, measures of spread, and data displays from more than one random sample to compare and draw conclusions about more than one population
- Use mean and MAD, or median and IQR, from random samples to assess whether the differences in the samples are due to natural variability or due to meaningful differences in the underlying populations

7-8 Samples and Populations: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Making Sense of Samples	Investigation 2 Choosing a Sample From a Population	Investigation 3 Using Samples to Draw Conclusions
Problem 1.1 Comparing Performances: Using Center and Spread FQ: Given a set of results, how might you use measures of center and variability (spread) to judge overall performance? Problem 1.2 Which Team Is Most Successful? Using the MAD to Compare Samples FQ: What strategies might you use to evaluate numerical outcomes and judge success? Problem 1.3 Pick Your Preference: Distinguishing Categorical Data From Numerical Data FQ: How might you compare results to see if each sample responded to a survey in a similar way? How can using percentages help you make comparisons? Problem 1.4 Are Steel-Frame Coasters Faster Than Wood-Frame Coasters? Using the IQR to Compare Samples FQ: How might you decide whether steel-frame coasters or wood-frame coasters are faster?	Problem 2.1 Asking About Honesty: Using a Sample to Draw Conclusions FQ: What is a population? What is a sample? What is a sampling plan? Problem 2.2 Selecting a Sample: Different Kinds of Samples FQ: How could you select a sample of your school population to survey? Problem 2.3 Choosing Random Samples: Comparing Samples Using Center and Spread FQ: How could you use statistics of a random sample of data to make predictions about an entire population? Problem 2.4 Growing Samples: What Size Sample to Use? FQ: Can you make good statistical estimates with less work by selecting smaller samples? How does sample size relate to the accuracy of statistical estimates?	Problem 3.1 Solving an Archeological Mystery: Comparing Samples Using Box Plots FQ: How might you analyze samples from known and unknown populations to determine whether the unknown population has one or more attributes in common with the known population? Problem 3.2 Comparing Heights of Basketball Players: Using Means and MADs FQ: How can you determine whether differences in sample data are large enough to be meaningful, or just due to naturally occurring variability from one sample to another? Problem 3.3 Five Chocolate Chips in Every Cookie: Using Sampling in a Simulation FQ: How can you simulate a real-world problem? How can you analyze the data that you collect from that simulation to draw conclusions? Problem 3.4 Estimating a Deer Population: Using Samples to Estimate the Size of a Population FQ: How can you estimate the size of a large population?
Mathematical Reflections 1a. A new term is used in this Investigation: sample. What do you think sample means? 1b. Suppose you have data from a 7th-grade class. The data are answers to the questions: • What is your favorite movie? • How many movies do you watch per week? i. Which statistic can you use to summarize the results of the data? ii. How could you use the data to predict the number of students in the entire 7th grade who would say they watch two movies per week? 2a. How do graphs of distributions help you compare data sets? 2b. How do measures of center help you compare data sets? 2c. How do measures of spread help you compare data sets? 3. When does it make sense to compare groups using counts, or frequencies? When does it make sense to compare groups using percents, or relative frequencies? Explain.	Mathematical Reflections 1. Why are data often collected from a sample rather than from an entire population? 2. Describe four plans for selecting a sample from a population. Discuss the advantages and disadvantages of each plan. 3a. How are random samples different from convenience, voluntary-response, and systematic samples? 3b. Why is random sampling preferable to the other sampling plans? 3c. Describing three plans for selecting a random sample from a given population. What are the advantages and disadvantages of each plan? 4. Suppose you select several random samples of size 30 from the same population. 4a. When you compare the samples to each other, what similarities and differences would you expect to find among the measures of center and spread? 4b. When you compare the samples to the larger population, what similarities and differences would you expect to find among the measures of center and spread? 5. How has your idea of the term sample changed from what you wrote in Mathematical Reflections, Investigation 1?	Mathematical Reflections 1a. How can you use statistics to compare samples? How can you use samples to draw conclusions about the populations from which they are selected? 1b. In what ways might a data distribution for a sample be similar to or different from the data distribution for the entire population? 2a. How can you use box plots, medians, and IQRs to compare samples? Give an example. 2b. How can you use means and MADs to compare samples? Give an example. 2c. How can you use statistics to decide whether differences between samples are expected due to natural variability or reflect measureable differences in underlying populations? 3a. How can you use simulations to generate samples? 3b. How can you use data from a capture-tag-recapture simulation to estimate the actual size of a population? 4. The process of statistical investigation involves posing questions, collecting and analyzing data, and making interpretations to answer the original questions. Choose a Problem from this Investigation. Explain how you used the process of statistical investigation to solve the Problem.

8-1: Thinking with Mathematical Models
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Linear and Nonlinear Relationships Recognize and model linear and nonlinear relationships in bivariate data
- Represent data patterns using graphs, tables, word descriptions and algebraic expressions
- Use mathematical models to answer questions about linear relationships
- Investigate the nature of linear variation in contexts
- Write linear functions from verbal, numerical, or graphical information
- Analyze, approximate, and solve linear equations
- Model situations with inequalities expressed as “at most” and “at least” situations
- Investigate the nature of inverse variation in contexts
- Use mathematical models to answer questions about inverse variation relationships
- Compare inverse variation relationships with linear relationships

Data Analysis Measure variation in data and strength of association in bivariate data
- Use data patterns to make predictions
- Fit a line to data that show a linear trend and measure goodness of fit
- Analyze scatter plots of bivariate data to determine the strength of the linear relationship between the two variables.
- Use correlation coefficients informally to describe the strength of the linear relationship illustrated by scatter plots.
- Distinguish between categorical and numerical variables.
- Use 2-way tables and analysis of cell frequencies and relative frequencies to help in deciding whether two categorical variables are related.
- Use standard deviation to measure variability in data distributions

8-1 Thinking with Mathematical Models: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Exploring Data Patterns	Investigation 2 Linear Models and Equations	Investigation 3 Inverse Variation	Investigation 4 Variability and Associations in Numerical Data	Investigation 5 Variability and Associations in Categorical Data
Problem 1.1 Bridge Thickness and Strength FQ: How would you describe the relationship between bridge strength and bridge thickness revealed by your experiment? Problem 1.2 Bridge Length and Strength FQ: How would you describe the pattern relating bridge strength to bridge length shown in your experimental data? Problem 1.3 Custom Construction Parts: Finding Patterns FQ: How can you predict if a pattern between variables will be linear or nonlinear?	Problem 2.1 Modeling Linear Data Patterns FQ: How can you find a linear function that is a good model for a set of data and then measure the accuracy of that model with residuals? Problem 2.2 Up and Down the Staircase: Exploring Slope FQ: How do you write an equation for a linear function if you are given a graph, a table, or two points? Problem 2.3 Tree Top Fun: Equations for Linear Functions FQ: What strategies do you use in writing equations for linear functions? Problem 2.4 Boat Rental Business: Solving Linear Equations FQ: What strategies do you find useful to find solutions for linear equations? Problem 2.5 Amusement Park or Movies: Intersecting Linear Models FQ: When the graphs of two linear functions intersect, what do the coordinates of that intersection point tell you?	Problem 3.1 Rectangles with Fixed Area FQ: When the product of two variables is some fixed number, what is the pattern of change and how is that pattern of change reflected in tables and graphs of the relationship? Problem 3.2 Distance, Speed and Time FQ: What examples using distance, rate, and time show one variable inversely related to another? Problem 3.3 Planning a Field Trip: Finding Individual Cost FQ: How does the cost per person change if a fixed total cost is split among an increasing number of individual payers? Problem 3.4 Modeling Data Patterns FQ: What pattern in a table or graph of data suggests an inverse variation model and what strategies can you use to find an equation model for that kind of function?	Problem 4.1 Vitruvian Man: Relating Body Measurements FQ: If you have data relating two variables, how can you check to see whether a linear model is a good fit? Problem 4.2 Older and Faster: Negative Correlations FQ: From the scatter plot, how do you know if a linear model fits the data? How do you know if there are outliers? How do you know if the relationship is negative or positive? Problem 4.3 Correlation Coefficients and Outliers FQ: What does a correlation coefficient of 1, 0, or −1 suggest to you about the relationship between two variables? Problem 4.4 Measuring Variability: Standard Deviation FQ: How do you calculate the standard deviation for a data distribution and what does that statistic tell about the distribution?	Problem 5.1 Wood or Steel? That's the Question FQ: What does a two-way table show you about preferences among groups? Problem 5.2 Politics of Girls and Boys: Analyzing Data in Two-Way Tables FQ: Suppose you have recorded the counts of different preferences by group in a two-way table. How can you use those counts, or percents from the counts, to decide if two groups have the same preferences or not? Problem 5.3 After-School Jobs and Homework: Working Backward: Setting up a Two-Way Table FQ: Suppose you have data about the same trait in two groups. How can you organize the data to compare and decide if the groups are the same or not relative to the trait?
Mathematical Reflections 1. You can represent a relationship between variables with a table, a graph, a description in words, or an equation. 1a. How can you decide whether a relationship is linear by studying the pattern in a data table? 1b. How can you decide whether a relationship is linear by studying the pattern in a graph? 1c. How can you decide whether a relationship is linear by studying the words used to describe the variables? 1d. How can you decide whether a relationship is linear by studying the equation that expresses the relationship in symbolic form? 2. What are the advantages and disadvantages in finding patterns and making predictions?	Mathematical Reflections 1. Why is it helpful to use a linear model for a set of data? 2. When does it make sense to choose a linear function to model a set of data? 3. How would you find the equation for a linear function in the following situations? 3a. You are given a description of the variables in words. 3b. You are given a table of values for the variables 3c. You are given a graph of sample data points 4. What strategies can you use to solve a linear equation such as 500 = 245 + 5x? 5. What kind of mathematical sentences express "at least" and "at most" questions about linear functions?	Mathematical Reflections 1. Suppose the relationship between variables x and y is an inverse variation. 1a. How do the values of y change as the values of x increase? 1b. Describe the trend in a graph of (x, y) values. 1c. Describe the equation that relates the values of x and y. 2. How is an inverse variation similar to a linear relationship? How is it different?	Mathematical Reflections 1. Think about the pattern of points you see in a scatter plot. 1a. What pattern would you expect when two variables are related by a linear model with positive slope? 1b. What pattern would you expect when two variables are related by a linear model with negative slope? 1c. What would you expect to see in a scatter plot when two variables are unrelated? 2. You assessed the accuracy of linear models. 2a. What do outliers on a scatter plot indicate? 2b. What can you learn from the errors of prediction or residuals? 2c. What do you know about a linear model from the correlation coefficient? 3. What does the standard deviation tell you about a set of data?	Mathematical Reflections 1. What are categorical variables and what do they measure? 2. Suppose a survey asked teenagers and adults whether or not the use text messaging. 2a. How could you arrange the data to compare the groups? 2b. How would you decide that the two groups - teenagers and adults - were different in their use of text messaging? 2c. Suppose that one analysis compared only the numbers in each group - teenage text messager, teenage non-text messager, adult text messager, and adult non-text messager. How might the analysis result in misleading conclusions?

8-2 Looking for Pythagoras
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Pythagorean Theorem Understand and apply the Pythagorean Theorem
- Develop strategies for finding the distance between two points on a coordinate grid
- Explain a proof of the Pythagorean Theorem
- Use the Pythagorean Theorem and its converse to solve a variety of problems.
- Use the Pythagorean Theorem to find the equation of a circle with its center located at the origin

Real Numbers Understand the set of real numbers consists of rational and irrational numbers
- Interpret square roots and cube roots of numbers by making use of their related geometric representations
- Relate the area of a square to the side length of the square
- Estimate the values of square roots
- Estimate the values of cube roots
- Relate the volume of a cube to the edge length of the cube
- Compare numbers that can be represented as fractions (rational numbers) to numbers that cannot be represented as fractions (irrational numbers) and recognize that the set of real numbers consists of rational and irrational numbers.
- Represent rational numbers as fractions and as terminating decimals or repeating decimals
- Recognize that irrational numbers cannot be represented as fractions and are nonterminating, nonrepeating decimals
- Recognize that the square root of a whole number that is not a square is irrational
- Locate irrational numbers on a number line
- Use and understand properties of rational and irrational numbers.

8-2 Looking for Pythagoras: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Coordinate Grids	Investigation 2 Squaring Off	Investigation 3 They Pythagorean Theorem	Investigation 4 Using the Pythagorean Theorem: Understanding Rational numbers	Investigation 5 Using the Pythagorean Theorem: Analyzing Triangles and Circles
Problem 1.1 Driving Around Euclid: Locating Points and Finding Distances FQ: How do driving distance between two coordinates relate to each other? Problem 1.2 Planning Parks: Shapes on a Coordinate Grid FQ: How do the coordinates of endpoints of a segment help draw other lines, which re parallel or perpendicular to the segment? Problem 1.3 Finding Areas FQ: How does knowing how to calculate areas of rectangles and triangles help n the calculation of irregular areas?	Problem 2.1 Looking for Squares FQ: How many different square areas are possible to draw using the dot grid as vertices? Why are some square areas not possible? Problem 2.2 Square Roots FQ: What does $\sqrt{x}$ mean? How does it relate to x? Problem 2.3 Using Squares to Find Lengths FQ: How can you find the distance between any two points on a grid? Problem 2.4 Cube Roots FQ: What does it mean to take the cube root of a number?	Problem 3.1 Discovering the Pythagorean Theorem FQ: You know the sum of the two shortest side lengths of a triangle must be greater than the third side length. Is there a similar relationship among the squares on the sides of a triangle? Is the relationship the same for all triangles? Problem 3.2 A Proof of the Pythagorean Theorem FQ: How can you prove that the relationship observed in Problem 3.1 will work for all right triangles? Problem 3.3 Finding Distances FQ: How can you find the distance between any two points on a plane? Problem 3.4 Measuring the Egyptian Way: Lengths That Form a Right Triangle FQ: If a triangle with side lengths a, b, and c satisfies the relationship $a^2 + b^2 = c^2$, is the triangle a right triangle?	Problem 4.1 Analyzing the Wheel of Theodorus: Square Roots on a Number Line FQ: Dan you find distances that are exact square roots of all whole numbers? Can you order square roots on a number line? Problem 4.2 Representing Fractions as Decimals FQ: Why can you represent every fraction as a repeating or terminating decimal? How can you predict which representations will repeat and which will terminate? Problem 4.3 Representing Decimals as Fractions FQ: Can you represent every repeating or terminating decimal as a fraction? Problem 4.4 Getting Real: Irrational Numbers FQ: Can you identify every number as either rational or irrational?	Problem 5.1 Stopping Sneaky Sally: Finding Side Lengths FQ: How can you use the Pythagorean Theorem to find distances in a geometric shape? Problem 5.2 Analyzing Triangles FQ: How do the lengths of the sides of a 30-60-90 triangle relate to each other? Problem 5.3 Analyzing Circles FQ: What is the relationship between the coordinates of a point (x, y) on a circle with a center at the origin?
Mathematical Reflections 1. In the city of Euclid, how does the driving distance form one place to another compare to the flying distance? 2. Suppose you know the coordinates of two landmarks in Euclid. How can you find the distance between the landmarks? 3. What are some strategies for finding areas of figures drawn on a grid?	Mathematical Reflections 1. Describe how you would find the length of a line segment connecting two dots on dot paper. Be sure to consider horizontal, vertical, and tilted segments. 2a. Explain what it means to find the square root of a number. 2b. Explain whether or not a number can have more than one square root. 3a. Explain what it means to find the cube root of a number. 3b. Explain whether or not a number can have more than one cube root.	Mathematical Reflections 1. Suppose you are given the lengths of two sides of a right triangle. Describe how you can find the length of the third side. 2. Suppose two points on a grid are not on the same horizontal or vertical line. Describe how you can use the Pythagorean Theorem to find the distance between the points without measuring. 3. How can you determine whether a triangle is a right triangle if you know only the lengths of its sides?	Mathematical Reflections 1. Give three examples of fractions with decimal representations that terminate. 2. Give three examples of fractions with decimal representations that repeat. 3. Give three examples of irrational numbers, including one irrational number greater than 5. 4. How can you determine whether you can write a given decimal as a fraction?	Mathematical Reflections 1. Give at least two examples of ways in which the Pythagorean Theorem can be useful. 2. Describe the special properties of a 30-60-90 triangle. 3. What information do you need to write the equation of a circle with the center at the origin?

8-3: Growing, Growing, Growing
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Exponential Functions Explore problem situations in which two or more variables have an exponential relationship to each other

- Identify situations that can be modeled with an exponential function
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation
- Represent an exponential function with a table, graph, or equation
- Make connections among the patterns of change in a table, graph, and equation of an exponential function
- Compare the growth/decay rate and growth/decay factor for an exponential function and recognize the role each plays in an exponential situation
- Identify the growth/decay factor and initial value in problem situations, tables, graphs, and equations that represent exponential functions
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function
- Determine the values of the independent and dependent variables from a table, graph, or equation of an exponential function
- Use an exponential equation to describe the graph and table of an exponential function
- Predict the y-intercept from an equation, graph, or table that represents an exponential function
- Interpret the information that the y-intercept of an exponential function represents
- Determine the effects of the growth (decay) factor and initial value for an exponential function on a graph of the function
- Solve problems about exponential growth and decay from a variety of different subject areas, including science and business, using an equation, table, or graph
- Observe that one exponential equation can model different contexts
- Compare exponential and linear functions

Equivalence Develop understanding of equivalent exponential expressions

- Write and interpret exponential expressions that represent the dependent variable in an exponential function
- Develop the rules for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation
- Write and interpret equivalent expressions using the rules for exponents and operations
- Solve problems that involve exponents, including scientific notation

8-3 Growing, Growing, Growing: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Exponential Growth	Investigation 2 Examining Growth Patterns	Investigation 3 Growth Factors and Growth Rates	Investigation 4 Exponential Decay	Investigation 5 Patterns with Exponents
Problem 1.1 Making Ballots: Introducing Exponential Functions FQ: What are the variables in this situation and how are they related? Problem 1.2 Requesting a Reward: Representing Exponential Functions FQ: In what ways are the relationships represented in a chessboard and ballot-cutting situations similar? Different? Problem 1.3 Making a New Offer: Growth Factors FQ: How does the growth pattern for an exponential function show up in a table, graph, or equation that represents the function and how does it compare to the growth pattern in a linear function?	Problem 2.1 Killer Plant Strikes Lake Victoria: y-intercepts Other Than 1 FQ: What information do you need to write an equation that represents an exponential function? Problem 2.2 Growing Mold: Interpreting Equations for Exponential Functions FQ: How is the growth factor and initial population for an exponential function represented in an equation that represents the function? Problem 2.3 Studying Snake Populations: Interpreting Graphs of Exponential Functions FQ: How is the growth factor and initial population for an exponential function represented in a graph that represents the function?	Problem 3.1 Reproducing Rabbits: Fractional Growth Patterns FQ: How is the growth factor in this Problem similar to that in the previous Problems? How is it different? Problem 3.2 Investing for the Future: Growth Rates FQ: How are the growth factor and growth rate for an exponential function related? When might you use each in an exponential growth pattern? Problem 3.3 Making a Difference: Connecting Growth Rate and Growth Factor FQ: How does the initial population affect the growth patterns in an exponential function?	Problem 4.1 Making Smaller Ballots: Introducing Exponential Decay FQ: How does the pattern of change in this situation compare to growth patterns you have studied in previous Problems? How does the difference show up in a table, graph, and equation? Problem 4.2 Fighting Fleas: Representing Exponential Decay FQ: How can you recognize an exponential decay function from a contextual setting, table, graph, and equation that represents the function? Problem 4.3 Cooling Water: Modeling Exponential Decay FQ: How can you find the initial population and decay factor for an exponential decay relationship?	Problem 5.1 Looking for Patterns Among Exponents FQ: What patterns did you observe in the table of powers? Problem 5.2 Rules of Exponents FQ: What are several rules for working with exponents and why do they work? Problem 5.3 Extending the Rules of Exponents FQ: How are the rules for integral exponents related to rational exponents? How are the rules for exponents useful in writing equivalent expressions with exponents? Problem 5.4 Operations with Scientific Notation FQ: How does scientific notation help to solve problems? Problem 5.5 Revisiting Exponential Functions FQ: What are the effects of *a* and *b* on the graph of $y = a(b^x)$, $b \neq 0$.
Mathematical Reflections 1. Describe an exponential growth pattern. Include key properties such as growth factors. 2. How are exponential functions similar to and different from the linear functions you worked with in earlier Units?	Mathematical Reflections 1. How can you use a table, a graph, and an equation that represent an exponential function to find the *y*-intercept and growth factor for the function? Explain. 2. How can you use the *y*-intercept and growth factor to write an equation that represents an exponential function? Explain. 3. How would you change your answers to Questions 1 and 2 for a linear function?	Mathematical Reflections 1. Suppose you know the initial value for a population and the yearly growth rate. 1a. How can you determine the population several years from now? 1b. How is a growth rate related to the growth factor for the population? 1c. How can you use this information to write an equation that models the situation? 2. Suppose you know the initial value for a population and the yearly growth factor. 2a. How can you determine the population several years from now? 2b. How can you determine the yearly growth rate? 3. Suppose you know the equation that represents the exponential function relating the population *p* and the number of years *n*. How can you determine the doubling time for the population?	Mathematical Reflections 1. How can you recognize an exponential decay pattern from the following? 1a. a table of data 1b. a graph 1c. an equation 2. How are exponential growth functions and exponential decay functions similar? How are they different? 3. How are exponential decay functions and decreasing linear functions similar? How are they different?	Mathematical Reflections 1a. Describe some of the rules for operating with exponents. 1b. What is scientific notation? What are its practical applications? 2. Describe the effects of *a* and *b* on the graph of $y = a(b^x)$. 3. Compare exponential and linear functions. Include in your comparison information about their patterns of change, *y*-intercepts, whether the function is decreasing or increasing, and any other information you think is important. Include examples of how they are useful.

8-4: Frogs, Fleas and Painted Cubes
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Quadratic Functions Explore problem situations in which two variables are in a quadratic relationship

- Identify situations that can be modeled by quadratic functions
- Identify the pattern of change between two variables that represent a quadratic function in a situation, table, graph, or equation
- Determine values of the independent and dependent variables in a quadratic function from a table, graph, or equation
- Represent a quadratic function with a table, graph, and equation
- Make connections among the equation of a quadratic function, its graph, and the patterns of change in its table
- Use a quadratic equation to describe the characteristics of its graph and table
- Determine whether a quadratic function will have a maximum or a minimum point and predict the maximum or minimum point from its equation, graph, or table
- Predict the x- and y-intercepts from the equation, graph, or table of a quadratic function
- Predict the line of symmetry from an equation, graph, or table of a quadratic function
- Interpret the information that the x- and y-intercepts and maximum or minimum point represent
- Use an equation, graph, and table to solve problems involving quadratic relationships
- Observe that one quadratic equation can model different contexts
- Compare linear, quadratic, and exponential functions

Equivalence Develop an understanding of equivalent quadratic expressions

- Write and interpret a quadratic expression to represent the dependent variable in a quadratic function
- Use an area model to develop an understanding of the Distributive Property
- Use the Distributive Property to write equivalent quadratic expressions in expanded or factored form
- Select and interpret the appropriate equivalent quadratic expression (in factored or expanded form) for predicting the x- and y-intercepts, maximum or minimum point, and the line of symmetry for a graph of a quadratic function

8-4 Frogs, Fleas, and Painted Cubes: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Introduction to Quadratic Functions	Investigation 2 Quadratic Expressions	Investigation 3 Quadratic Patterns of Change	Investigation 4 Frogs Meet Fleas on a Cube: More Applications of Quadratic Functions
Problem 1.1 Staking a Claim: Maximizing Area FQ: Describe the shape of a graph that represents the areas of rectangles with a fixed perimeter. Problem 1.2 Reading Graphs and Tables FQ: How does the maximum area of rectangles with a fixed perimeter appear in a graph or a table? Problem 1.3 Writing an Equation FQ: How can you write an equation for the areas of rectangles with a fixed perimeter?	Problem 2.1 Trading Land: Representing Areas of Rectangles FQ: If the length *n* of a square is increased by 2 units and its width *n* decreased by 2 units, what two equivalent expressions represent the area of the new figure? Problem 2.2 Changing Dimensions: The Distributive Property FQ: How does the Distributive Property apply to quadratic expressions? Explain. Problem 2.3 Factoring Quadratic Expressions FQ: What is a method for factoring an expression as a product of two or more factors? How is this related to the Distributive Property? Problem 2.4 Quadratic Functions and Their Graphs FQ: How can you use a quadratic equation to predict the x- and y-intercepts, maximum/minimum points, and line of symmetry of its graph?	Problem 3.1 Exploring Triangular Numbers FQ: How many dots (or squares) are in the n^{th} triangular number? Problem 3.2 Counting Handshakes: Another Quadratic Function FQ: If each team has *n* members, how many handshakes will occur? Problem 3.3 Examining Patterns of Change FQ: Describe the pattern of change between the number of people on a team and the number of handshakes that occur. Problem 3.4 Quadratic Functions and Patterns of Change FQ: Compare the pattern of change for a quadratic function to the patterns of change for linear and exponential functions.	Problem 4.1 Tracking a Ball: Interpreting a Table and an Equation FQ: How can you predict the maximum height of a ball from the graph of a quadratic function? Problem 4.2 Measuring Jumps: Comparing Quadratic Functions FQ: How can you predict the y-intercept of a quadratic function from its graph, table, or equation? Problem 4.3 Painted Cubes: Looking at Several Functions FQ: When a painted cube with edge length *n* is separated into n^3 small cubes, how many of these cubes will have paint on three faces? Two faces? One face? No faces? Problem 4.4 Putting It All Together: Comparing Functions FQ: What can you learn about a function from a table, graph, or equation that represents the function?
Mathematical Reflections 1a. Describe the characteristics of graphs and tables of quadratic functions you have observed so far. 1b. How do the patterns in a graph of a quadratic function appear in the table of values for the function? 2. Describe two ways to find the maximum area for rectangles with a fixed perimeter. 3. How are tables, graphs, and equations for quadratic functions different from those for linear and exponential functions?	Mathematical Reflections 1. Explain how you can use the Distributive Property to answer each question. Use examples to help with your explanations. 1a. Suppose a quadratic expression is in factored form. How can you find an equivalent expression in expanded form? 1b. Suppose a quadratic expression is in expanded form. How can you find an equivalent expression in factored form? 2. Describe what you know about the shape of the graph of a quadratic function. Include important features of the graph and describe how you can predict these features from the equation of the function.	Mathematical Reflections 1a. In what ways is the triangular-number relationship similar to the relationships in the handshake problems? In what ways are these relationships different? 1b. In what ways are the quadratic functions in this Investigation similar to the quadratic functions in Investigations 1 and 2? In what ways are they different? 2a. In a table of values for a quadratic function, how can you use the pattern of change to predict the next value? 2b. How can you use a table of values to decide if a function is quadratic? 3. Compare the patterns of change for linear, exponential, and quadratic functions.	Mathematical Reflections 1. Describe three real-world situations that can be modeled by quadratic functions. For each situation, give examples of questions that quadratic representations help to answer. 2. How can you recognize a quadratic function from 2a. a table? 2b. a graph? 2c. an equation? 3. What clues in a problem situation indicate that a linear, exponential, or quadratic function is an appropriate model for the data in the problem?

8-5: Butterflies, Pinwheels, and Wallpaper
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Transformations Describe types of transformations that relate points by the motions of Reflections, rotations, and translations, and describe methods for identifying and creating symmetric plane figures

- Recognize properties of Reflections, rotation, and translation transformations
- Explore techniques for using rigid motion transformations to create symmetric designs
- Use coordinate rules for basic rigid motion transformations

Congruence and Similarity Understand congruence and similarity and explore necessary and sufficient conditions for establishing congruent and similar shapes

- Recognize that two figures are congruent if one is derived from the other by a sequence of Reflections, rotation, and/or translation transformations
- Recognize that two figures are similar if one can be obtained from the other by a sequence of Reflections, rotations, translations, and/or dilations
- Use transformations to describe a sequence that exhibits the congruence between figures
- Use transformations to explore minimum measurement conditions for establishing congruence of triangles
- Use transformations to explore minimum measurement conditions for establishing similarity of triangles
- Relate properties of angles formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations
- Use properties of congruent and similar triangles to solve problems about shapes and measurements

8-5 Butterflies, Pinwheels and Wallpaper: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Symmetry and Transformations	Investigation 2 Transformations and Congruence	Investigation 3 Transforming Coordinates	Investigation 4 Dilations and Similar Figures
Problem 1.1 Butterfly Symmetry: Line Reflections FQ: What does it mean to say that a figure has Reflections or flip symmetry? How is each point related to its image under transformation by Reflections in a line? Problem 1.2 In a Spin: Rotations FQ: What does it mean to say that a figure has rotation or turn symmetry? How is each point related to its image under transformation by rotation? Problem 1.3 Sliding Around: Translations FQ: What does it mean to say that a figure has translation or slide symmetry? How is each point related to its image under transformation by translation? Problem 1.4 Properties of Transformations FQ: How, if at all, will the shape, size, and position of a geometric figure change after each of the transformations studied in this investigation – flip, turn, or slide?	Problem 2.1 Connecting Congruent Polygons FQ: What does it mean to say two geometric shapes are congruent to each other and how could you demonstrate congruence with movable copies of the figures? Problem 2.2 Supporting the World: Congruent Triangles I FQ: How much information do you need to decide that two triangles are probably congruent or not congruent? How do you go about planning transformations that ‘move’ one triangle onto another? Problem 2.3 Minimum Measurement: Congruent Triangles II FQ: What is the smallest number of side and angle measurements that will allow you to conclude that two triangles are congruent?	Problem 3.1 Flipping on a Grid: Coordinate Rules for Reflections FQ: How can you describe the ‘motions’ of points under Reflections with coordinate rules in the form (x,y)→(□,□) tells how to ‘move’ any point to its image under a translation? Problem 3.2 Sliding on a Grid: Coordinate Rules for Translations FQ: What kind of coordinate rule (x,y)→(□,□) tells how to ‘move any point to its image under a translation? Problem 3.3 Spinning on a Grid: Coordinate Rules for Rotations FQ: What are the coordinate rules that describe ‘motion’ of points on a grid under turns of 90° and 180°? Problem 3.4 A Special Property of Translations and Half-Turns FQ: How are lines and their images under translations and half-turns related to each other? Problem 3.5 Parallel Lines, Transversals, and Angle Sums FQ: When two parallel lines are cut by a transversal, what can be said about the angles formed? What is always true about the angle measures in a triangle? How do you know that your answers are correct?	Problem 4.1 Focus on Dilations FQ: What coordinate rules model dilations and how do dilations change or preserve characteristics of the original figure? Problem 4.2 Return of Super Sleuth: Similarity Transformations FQ: How can you use transformations to check whether two figures are similar or not? Problem 4.3 Checking Similarity Without Transformations FQ: What information about the sides and angles of two triangles will guarantee that they are similar? Problem 4.4 Using Similar Triangles FQ: What facts about similar triangles allow you to find lengths in very large figures even when they can’t be reached to measure?
Mathematical Reflections 1. How would you explain to someone how to make a design with: 1a. Reflectionsal symmetry? 1b. rotational symmetry? 1c. translational symmetry? 2. How are points and their images related by each of these geometric transformations? 2a. Reflections in line m 2b. rotation of d° about point P 2c. translation with distance and direction set by the segment from point X to point X’. 3. How do Reflections, rotations, and translations change the size and shape of line segments, angles, and/or polygons, if at all?	Mathematical Reflections 1. How can you find a sequence of flips, turns, and slides to “move” one figure exactly onto another to show that they are congruent? 2. What information about the sides and angles of two triangles will guarantee you can “move” one triangle onto the other? 3. How could you convince someone that two given triangles are not congruent?	Mathematical Reflections 1. What are the general forms of the coordinate rules for these transformations? 1a. Reflections in the y-axis 1b. refection in the x-axis 1c. counterclockwise rotation of 90° about the origin 1d. counterclockwise rotation of 180° about the origin 1e. translation that “moves” points a units horizontally and b units vertically 2. What is the effect of translation and half-turns on lines? 3. How has your knowledge of transformations changed or extended what you already knew about the angles formed by two parallel lines and a transversal? 4. How has your knowledge of transformations changed or extended what you already knew about the sum of the angle measures of a triangle?	Mathematical Reflections 1. How would you explain what it means for two geometric shapes to be similar using 1a. everyday words that most people could understand? 1b. technical terms of mathematics? 2a. Suppose you dilate a polygon to form a figure of a different size. How will the side lengths, angle measures, perimeters, areas, and slopes of the sides of the two figures be alike? How will they be different? 2b. How has your knowledge of dilations changed or extended what you already knew about similarity. 3. What is the least amount of information you need in order to be sure that two triangles are similar? 4. How do you use similarity to find the side lengths of similar figures?

8-6: Say It With Symbols
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Equivalence Develop understanding of equivalent expressions and equations
- Model situations with symbolic statements
- Recognize when two or more symbolic statements represent the same context
- Use the properties of real numbers, such as the Distributive Property, to write equivalent expressions
- Determine if different symbolic expressions are mathematically equivalent
- Interpret the information that equivalent expressions represent in a given context
- Determine the equivalent expression or equation that is most helpful in answering a particular question about a relationship
- Use algebraic equations to describe the relationship among the volumes of cylinders, cones and spheres that have the same height and radius
- Solve linear equations involving parentheses
- Determine if a linear equation has a finite number of solutions, an infinite number of solutions, or no solution
- Develop understanding and some fluency with factoring quadratic expressions
- Solve quadratic equations by factoring
- Recognize how and when to use symbols, rather than tables or graphs, to display relationships, generalizations, and proofs

Functions Develop an understanding of specific functions such as linear, exponential and quadratic functions
- Develop proficiency in identifying and representing relationships expressed in problem contexts with appropriate functions and use these relationships to solve the problem
- Analyze equations to determine the patterns of change in the tables and graphs that the equations represent
- Relate parts of a symbolic statement or expression to the underlying properties of the relationship they represent and to the context of the problem
- Determine characteristics of a graph (intercepts, maxima and minima, shape, etc.) of an equation by looking at its symbolic representation

8-6 Say It With Symbols: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Making Sense of Symbols: Equivalent Expressions	Investigation 2 Combining Expressions	Investigation 3 Solving Equations	Investigation 4 Looking Back at Functions	Investigation 5 Reasoning with Symbols
Problem 1.1 Tiling Pools: Writing Equivalent Expressions FQ: What expression(s) represents the number of border tiles needed to surround a square pool with side length *s*? Problem 1.2 Thinking in Different Ways: Determining Equivalence FQ: How can you determine if two or more expressions are equivalent? Problem 1.3 The Community Pool Problem: Interpreting Expressions FQ: What information goes an expression represent in a given context? Problem 1.4 Diving In: Revisiting the Distributive Property FQ: What information does an expression represent in a given context?	Problem 2.1 Walking Together: Adding Expressions FQ: What are the advantages and disadvantages of using one equation rather than two or more equations to represent a situation? Problem 2.2 Predicting Profit: Substituting Expressions FQ: What are some ways that you can combine one or more expressions (or equations) to create a new expression (or equation)? Problem 2.3 Making Candles: Volumes of Cylinders, Cones, and Spheres FQ: What equations represent the relationships among the volumes of cylinders, cones, and spheres? Problem 2.4 Selling Ice Cream: Solving Volume Problems FQ: What formulas are useful in solving problems involving volumes of cylinders, cones, and spheres?	Problem 3.1 Selling Greeting Cards: Solving Linear Equations FQ: What strategies can you use to solve equations that contain parentheses? Problem 3.2 Comparing Costs: Solving More Linear Equations FQ: What are strategies for finding a solution that is common to two-variable linear equations? Problem 3.3 Factoring Quadratic Equations FQ: What are some strategies for factoring a quadratic expression? Problem 3.4 Solving Quadratic Equations FQ: What are some strategies for solving quadratic equations?	Problem 4.1 Pumping Water: Looking at Patterns of Change FQ: How can you use an equation to answer particular questions about a function and the situation it represents? Problem 4.2 Area and Profit – What's the Connection? Using Equations FQ: How can two different contexts be represented by the same equation? Problem 4.3 Generating Patterns: Linear, Exponential, Quadratic FQ: How can you determine the patterns of change of a function from a table of data for the function? Problem 4.4 What's the Function? Modeling With Functions FQ: How can you determine which function to use to solve or represent a problem?	Problem 5.1 Using Algebra to Solve a Puzzle FQ: How can you determine to use to solve or represent a problem? Problem 5.2 Odd and Even Revisited FQ: How can you use algebra to represent and prove a conjecture about numbers? Problem 5.3 Squaring Odd Numbers FQ: What are some strategies for making and proving a conjecture?
Mathematical Reflections 1. What does it mean to say that two expressions are equivalent? 2. Explain how you can use the Distributive Property to write equivalent expressions. 3. Explain how you can use the Distributive and Commutative properties to show that two or more expressions are equivalent.	Mathematical Reflections 1. Describe a situation in which it is helpful to add expressions to form a new expression. Explain how you can combine the expressions. 2. Describe a situation in which it is helpful to substitute an equivalent expression for a quantity in an equation. 3. What are the advantages and disadvantages of working with one equation rather than two or more equations in a given situation? 4. Write an expression that represents the volume of each three-dimensional figure. Explain your reasoning. 4a. cylinder 4b. cone 4c. sphere	Mathematical Reflections 1a. Describe some general strategies for solving linear equations, including those with parentheses. Give examples that illustrate your strategies. 1b. Describe how you can tell if a linear equation has a finite number of solutions, an infinite number of solutions, or no solutions. 2. Describe some strategies for solving quadratic equations of the form $ax^2 + bx + c = 0$. Give examples. 3. How are the solutions of linear and quadratic equations related to graphs of the equations?	Mathematical Reflections 1. Describe how you can tell whether an equation is a linear, an exponential, or a quadratic function. 2. Describe how you can determine specific features of the graph of a function from its equation. Include its shape, x- and y-intercepts, maximum and minimum points, and patterns of change. 3. Describe how you can recognize which function to use to solve an applied problem.	Mathematical Reflections 1. Describe how and why you could use symbolic statements to represent relationships and conjectures. 2. Describe how you can show that your conjectures are correct.

8-7: It's In The System

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Linear Equations Develop understanding of linear equations and systems of linear equations

- Recognize linear equations in two variables in standard form $Ax+By=C$
- Recognize that a linear equation in the form $Ax+By=C$ has infinitely many solutions (x,y) and the graph of those solutions is always a straight line
- Recognize that the form $Ax+By=C$ of linear equations is equivalent to the form $y=mx+b$ for linear equations
- Continue to develop skills in solving a linear equation in two variables by graphing and with algebraic methods
- Recognize that solving a system of linear equations is equivalent to finding values of the variables that will simultaneously satisfy all equations in the system
- Develop skills in solving systems of linear equations by graphing solutions of separate equations; writing the system of equations in equivalent $y=mx+b$form; or using combinations of the system to eliminate one variable
- Recognize that systems of linear equations in the form $\begin{cases} Ax+By=C \\ Dx+Ey=F \end{cases}$ may have exactly one solution, which is the intersection point of the lines represented by the equations; infinitely many solutions, which is represented by a single line for both equations; or no solution, which is represented by two parallel lines
- Choose between graphing and symbolic methods to efficiently find the solution to a particular system of linear equations
- Gain fluency with symbol manipulation in solving systems of linear equations
- Solve problems that involve systems of linear equations

Linear Inequalities Develop understanding of graphing and symbolic methods for solving linear inequalities with one and two variables

- Recognize differences between strict and inclusive inequalities
- Continue to develop skill in solving a linear inequality in two variables by graphing and symbolic methods
- Develop skill in solving systems of linear inequalities by graphing solutions of each inequality and finding the region of feasible points that satisfy both inequalities; and solving inequalities to find pairs of numbers that satisfy both inequalities
- Choose between graphing and symbolic methods to efficiently find the region of feasible points to a particular system of linear inequalities
- Solve a simple system consisting of a linear equation and a quadratic equation in two variables symbolically and graphically
- Solve problems that involve linear inequalities or systems of linear inequalities

8-7 It's in the System: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Linear Equations With Two Variables	Investigation 2 Solving Linear Systems Symbolically	Investigation 3 Systems of Functions and Inequalities	Investigation 4 Systems of Linear Inequalities
Problem 1.1 Shirts and Caps: Solving Equations With Two Variables FQ: What kind of solution will be found for an equation like $3x + 5y = 13$ with two variables? What will the graphs of those two solutions look like? Problem 1.2 Connecting $Ax + By = C$ and $y = mx + b$ FQ: How can one change an equation from $Ax + By = C$ form to an equivalent $y = mx + b$ form and vice versa? Problem 1.3 Booster Club Members: Intersecting Lines FQ: What happens when you search for common solutions to two linear equations with two variables?	Problem 2.1 Shirts and Caps Again: Solving Systems With $y = mx +b$ FQ: How can you solve a system of two linear equations with two variables by writing each equation in equivalent $y = mx + b$ form? What are the solution possibilities for such systems and how are they shown by graphs of the solutions? Problem 2.2 Taco Truck Lunch: Solving System by Combining Equations I FQ: How can you solve a system of linear equations by combining the two equations into one simpler equation by addition or subtraction? Problem 2.3 Solving Systems by Combining Equations II FQ: How can equations in a system be transformed to equivalent forms that make it easier to solve by combination to eliminate variables?	Problem 3.1 Comparing Security Services: Linear Inequalities FQ: How can you use function graphs to find the solutions of an inequality like $ax + b < cx + d$? How can the solutions be represented on a number line graph? Problem 3.2 Solving Linear Inequalities Symbolically FQ: How does applying the same operation to both sides of an inequality change the relationship of the two quantities being compared (or not)? How can linear inequalities be solved by strategies that are very similar to strategies for solving linear equations? Problem 3.3 Operating at a Profit: Systems of Lines and Curves FQ: What are the possible solutions for a system that includes one linear and one quadratic function and how can you find these solutions?	Problem 4.1 Limited Driving Miles: Inequalities With Two Variables FQ: If a problem involves solving an inequality like $ax + by \le c$, how many solutions would you expect to find and what would a coordinate graph of those solutions look like? Problem 4.2 What Makes a Car Green: Solving Inequalities by Graphing I FQ: What graph of solutions (in the first quadrant) would you expect for an inequality with the general form $ax + by \le c$? Problem 4.3 Feasible Points: Solving Inequalities by Graphing II FQ: What graph of solutions would you expect for an inequality with the general form $ax +by \le c$? Problem 4.4 Miles of Emissions: Systems of Linear Inequalities FQ: What do you look for to solve a system of linear inequalities and what will the graph of a solution look like?
Mathematical Reflections 1. What pattern will result from plotting all points (x,y) that satisfy an equation in the form $Ax + By = C$? 2. How can you change linear equations in the form $Ax + By = C$ to $y = mx + b$ form and vice versa? Explain when one form might be more useful than the other. 3. How can you use a graph to find values of x and y that satisfy systems of two linear equations in the form $Ax + By = C$?	Mathematical Reflections 1. What is the goal in solving a system of linear equations? 2. What strategies can you use to solve a system of linear equations? 3. How can you check a possible solution of a system of linear equations?	Mathematical Reflections 1. How can you use coordinate graphs to solve linear equations such as $ax + b = cx + d$ and linear inequalities such as $ax + b < cx + d$? 2. How can you use symbolic reasoning to solve inequalities such as $ax + b < cx + d$? 3. What strategies can you use to solve systems of equations and inequalities that involve linear and quadratic functions or lines and circles?	Mathematical Reflections 1. Suppose you are given one linear inequality with two variables. How could you use a graph to find solutions of the inequality? 2. Suppose you were given a system of two linear inequalities. How could you use a graph to find solutions of the system?

8-8: Function Junction
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Functions Understand equivalence of algebraic expressions and functions

- Describe domain and range of functions
- Use $f(x)$ notation to describe and operate with functions
- Construct and interpret inverses of functions
- Analyze function rates of change using graphs
- Identify contexts and graphs of step and piecewise defined functions
- Analyze polynomial functions and their graphs
- Identify, analyze, and solve problems related to arithmetic and geometric sequences
- Compare arithmetic and geometric sequences to linear and exponential functions
- Recognize and solve problems using special kinds of functions

Equivalence Understand equivalence of algebraic expressions and functions

- Connect expressions for functions whose graphs are related by translation and/or stretching
- Develop and use vertex form to graph quadratic functions and solve quadratic equations
- Connect polynomial expressions and graphs of the polynomial functions they define, in order to identify max/min points, intercepts, and solutions of equations
- Use completing the square to write quadratics in equivalent vertex form
- Develop the quadratic formula for solving equations
- Develop complex numbers and operations
- Develop algorithms for adding, subtracting, and multiplying polynomials

8-8 Function Junction: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 The Families of Functions	Investigation 2 Arithmetic and Geometric Sequences	Investigation 3 Transforming Graphs, Expressions, and Functions	Investigation 4 Solving Quadratic Equations Algebraically: Completing the Square and Using the Quadratic Formula	Investigation 5 Polynomial Expressions, Functions, and Equations
Problem 1.1 Filling Functions FQ: How does the shape of a function graph tell the rate of change in the dependent variable as the independent variable changes? Problem 1.2 Domain, Range, and Function Notation FQ: What do the terms domain and range tell about a function, and how is f(x) notation used to represent a function? Problem 1.3 Taxi Fares, Time Payments, and Step Functions FQ: What patterns of change can be modeled by functions called step-functions? Problem 1.4 Piecewise defined functions FQ: What patterns of change can be modeled by functions called piecewise defined? Problem 1.5 Inverse Functions FQ: What makes one function g(x) the inverse of another function f(x)? How can you find the inverse of a function f(x)?	Problem 2.1 Arithmetic Sequences FQ: What are the defining properties of an arithmetic sequence? Problem 2.2 Geometric Sequences FQ: What are the defining properties of a geometric sequence?	Problem 3.1 Sliding Up and Down: Vertical Translation of Functions FQ: If graphs of functions are related by sliding up and down, how are the expressions related? Problem 3.2 Stretching and Flipping Up and Down: Multiplicative Transformations of Functions FQ: If graphs of functions are related by stretching away from or towards the x-axis and/or reflecting across that axis, how are the expressions related? Problem 3.3 Sliding Left and Right: Horizontal Translations of Functions FQ: If graphs of functions are related by sliding left or right, how are the expressions related? Problem 3.4 Horizontal Translations of Functions FQ: How can you use the algebraic expression for any quadratic function f(x) = $a(x \pm b)^2 \pm c$ to predict the shape and location of the graph?	Problem 4.1 Solving Quadratic Equations Algebraically FQ: What strategies allow you to solve quadratic equations algebraically, and how are the algebraic and graphical solutions related to each other? Problem 4.2 Completing the Square FQ: How can a quadratic expression be written in equivalent vertex form? How does this help solve any quadratic equation? Why is the process of re-writing in vertex form called completing the square? Problem 4.3 The Quadratic Formula FQ: What is the Quadratic Formula, and how do you use it to solve any equation in the form $q(x) = ax^2 + bx + c = 0$ Problem 4.4 Complex Numbers FQ: How can the real number system be extended to a larger system that includes solutions for all quadratic equations?	Problem 5.1 Properties of Polynomial Expressions and Functions FQ: What are the patterns of change associated with polynomial expressions and functions of degree 2, 3, and 4, and how are those patterns shown in graphs? Problem 5.2 Combining Profits: Operating with Polynomials I FQ: How are the sum and difference of two polynomials calculated? Problem 5.3 Product Time: Operating with Polynomials II FQ: How is the product of two polynomials calculated? Problem 5.4 The Factor Game Revisited FQ: How has your understanding of factors (and products) changed since you last played the factor (and product) game? What ideas about whole number factors are similar to ideas about polynomial factors?
Mathematical Reflections 1. This investigation was about functions and the ways mathematicians think and write about them. 1a. What is a function? 1b. What are the domain and range of a function? 1c. What does a statement such as f(6) = 23 say about the function f(x)? 2a. What is a step function? 2b. Describe what graphs of step functions look like. 3a. What is a piecewise defined function? 3b. Give an example to illustrate this idea. 4a. When are two functions inverses of each other? 4b. What example would you give to illustrate this idea?	Mathematical Reflections 1a. Describe the defining properties of an arithmetic sequence? 1b. What examples would you give to illustrate the idea for someone? 2a. Describe the defining properties of a geometric sequence. 2b. What examples would you give to illustrate the idea for someone? 3. How are arithmetic and geometric sequences related to linear and exponential functions?	Mathematical Reflections 1. How will the rule for a function f(x) change if the graph is: 1a. Translated up or down by k? 1b. Stretched away from or toward the x-axis by a factor of k? 1c. Translated left or right by k? 2. How does the vertex form of a quadratic equation like $f(x) = (x - h)^2 + k$ (where h and k are positive numbers) help to sketch the graph of a function?	Mathematical Reflections 1. What are the key steps in writing a quadratic expression like $x^2 + 6x + 11$ in vertex form? 2. How does the Quadratic Formula help to solve equations in the form $ax^2 + bx + c = 0$? 3. What methods do you have for solving quadratic equations other than the Quadratic Formula? 4. What are the complex numbers? How are they added, subtracted, and multiplied?	Mathematical Reflections 1. What are polynomial expressions and functions? 2. How can one analyze the graph of a polynomial function p(x) to discover 2a. solutions to the equations p(x) = 0 2b. intervals on which values of the function are increasing or decreasing? 2c. points that show relative maximum or minimum values of the function? 3. What strategies give standard polynomial expressions for 3a. the sum or difference of two polynomials? 3b. the product of two polynomials?