Filling & Wrapping

Investigation 1

Standards:

7.RP.A.2 Recognize and represent proportional relationships between quantities. *Problem 4*

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *Problems 1, 2, and 3*

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. *Problem 4*

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. *Problems 1, 2, 3, and 4*

Objectives:

This Investigation has four Problems. The first Problem aims to diagnose and review student understanding of surface area and volume of a rectangular prism, which was developed in the Grade 6 Unit *Covering and Surrounding*. It reviews basic terms for describing prisms, such as vertices, faces, and edges, and it reviews the formula for calculating the volume of any rectangular prism. The second and third Problems use the challenge of minimizing packaging cost to extend student understanding of volume and surface area calculation for rectangular prisms. Then, in the fourth Problem, students explore the effects of scaling on the surface area and volume of a rectangular prism.

Investigation 2

Standards:

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. *Problem 2*

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. *Problem 3*

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. *Problems 1 and 2*

Objectives:

This Investigation has three Problems. The first Problem engages students in hands-on experimentation to discover interesting relationships between the surface area and volume of different prisms—if the height of the prism and the perimeter of its base are constant, the figure whose base has the most sides will have the greatest surface area and greatest volume. For most

of us, this is a counterintuitive result. However, it is similar to what students discovered when they compared areas of rectangles with fixed perimeter but different dimensions. It is surprising to students that the same piece of stiff paper can be folded in different ways to give solids with different volumes.

The second Problem guides students to generalize the layering strategy they used for finding volume of rectangular prisms to a strategy for finding volumes of other prisms. The general principle is that all such volumes can be calculated as the product of the height and the area of the base.

In the third Problem, students visualize the shapes (both solid and surface) that are formed when rectangular prisms are sliced by planar "knives."

Investigation 3

Standards:

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. *Problem 2*

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. *Problems 2 and 4*

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *Problems 1, 2, 3, and 4*

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. *Problems 1, 2, 3, and 4*

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. *Problems 2, 3, and 4*

Objective:

This Investigation has four Problems, all focused on developing student understanding of the relationships among circle radius (and diameter) and circumference and area. The first Problem asks students to explore the ratio of radius and diameter to circumference for a collection of circular objects, leading to a conjectured formula for those calculations. The second Problem engages students in some informal exploration of circle area by counting grid squares that fall within a sample of circles drawn on grid paper. The third Problem asks students to estimate the number of radius squares needed to cover a circle, leading to the informal connection of π to area calculation. The fourth Problem uses a well-known dissection argument to connect circumferences and areas of circles by showing how you can reassemble sectors of a circle into a shape that is almost a parallelogram.

The motivation for considering circles in a Unit on solid figures is the essential role of circles in defining and measuring cylinders. Students will develop formulas for surface areas and volumes

of cylinders in the next Investigation. Because cylinders are closely related to prisms, this reinforces what students have learned about solid figures. It also gives students an opportunity to apply their knowledge of surface areas and volumes of prisms to other figures.

Investigation 4

Standards:

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. *Problems 1, 2, 3, 4, and 5*

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. *Problems 1, 2, and 3*

7.G.B.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. *Problems 1, 2, 3, and 4*

Objective:

The Problems in this Investigation build naturally on students' understanding of surface area and volume of a prism, and extend these understandings to surface area and volume of a cylinder. They generalize from particular cylinders to finding formulas for the volume and surface area of any cylinder. Relationships among volumes of cylinders, spheres, and cones lead them to construct formulas for these other figures. This Investigation gives students opportunities to apply knowledge that addresses Grade 7 Common Core State Standards. This Unit also provides a foundation for related work in Grade 8 in *Say It With Symbols*.

Students first connect their knowledge of nets for prisms to analyze nets for cylinders. They derive a formula for surface area of a cylinder by using what they know about the circumference and area of a circle. The connection between prisms and cylinders is explored as students compare the volume of a cylinder to the volumes of prisms with the same lateral area but different bases. They see that the volume of a cylinder can be calculated using a familiar principle—multiplying the area of the base by the height. This reinforces their understanding of the volumes of prisms and again relies on knowledge about the area of a circle.

Comparing the surface areas of a prism and cylinder with the same volume pushes students to think about how surface area might vary and whether there is an optimal design for a given volume. Then students explore the relationships between the volumes of cylinders, spheres, and cones. They develop and apply formulas for the volume of a sphere and the volume of a cone.